

# Cognitive Biases, Ambiguity Aversion and Asset Pricing in Financial Markets\*

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## ABSTRACT

The behavior of agents in financial markets often displays biases or errors; for example, agents frequently do not compute probabilities correctly. However, we argue that these biases/errors are not always reflected in prices. In particular, we hypothesize that agents who make errors in computing probabilities lose confidence in their probability estimates when they face market prices that are inconsistent with their calculations; they then perceive (relevant) uncertain events as ambiguous (rather than risky), and hedge against this perceived ambiguity by holding a portfolio that generates unambiguous returns. These agents are price insensitive – they do not adjust their portfolio with changes in prices – and do not (directly) influence market prices. We identify price insensitive agents in an asset market experiment, and we test implications of our hypothesis: (i) agents who do not update correctly hold more balanced portfolios; (ii) the difference between portfolio holdings of agents who do not update correctly and agents who do update correctly increases as aggregate risk in the economy increases; (iii) prices are determined by the behavior of agents who do update correctly. Our experiments confirm all these hypotheses, but the extent to which observed prices conform to theoretical predictions decreases as the number of agents who do not update correctly increases. These observations reinforce our view that market prices trigger behavior that is consistent with ambiguity aversion.

**JEL Classification:** G11, G12, G14

**Keywords:** Asset pricing, ambiguity aversion, cognitive bias, Bayesian updating, market experiments.

# I. Introduction

Classical asset pricing models assume that investors are fully rational expected utility maximizers. Behavioral theories relax these assumptions in an effort to explain observed anomalies (deviations from the predictions of classical models). Typical behavioral models of asset pricing assume that a representative investor deviates from the decision predicted by the rational expected utility paradigm as the result of some particular form of errors (bounded rationality) or behavioral/cognitive biases and these errors/biases are reflected in the prices (Barberis, Shleifer and Vishny 1998; Daniel, Hirshleifer and Subrahmanyam 1998, 2001; Rabin and Vayanos 2009). While such behavioral models provide appealing explanations for some observed anomalies, they have also met with substantial skepticism (Brav and Heaton 2002).

One of the reasons for the skepticism about behavioral models is that there seems no reason to assume that *all* investors make the *same* errors or display the *same* biases, and it is not clear how heterogeneous errors and/or heterogeneous biases would be reflected in pricing. This paper argues that heterogeneous errors/biases may be reflected in prices in a very complicated way – or not at all.

The particular error/bias on which we focus here is improper Bayesian updating of prior beliefs. That many individuals make errors in Bayesian updating has been confirmed in numerous experiments (Kahneman and Tversky 1973; Grether 1992; El-Gamal and Grether 1995; Holt and Smith 2009), but these experiments also confirm that the magnitude (and even the kind) of these errors is quite heterogeneous across the population (of potential investors). One might expect that this heterogeneity would be reflected in asset prices. However, this expectation assumes that prices are affected by the behavior of investors who do not update correctly. We use experimental evidence and a theoretical model to argue that this may not be the case: investors who do not update correctly *and who know or suspect that they may not update correctly* may view the financial prospects as *ambiguous* rather than simply *risky*; if they are sufficiently averse to ambiguity they may choose not to be exposed to it at all, so that their beliefs will not be reflected in prices.<sup>1</sup>

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<sup>1</sup>The point that individuals make choices in a way that does not reflect beliefs or preferences has been made in quite a different experimental context by Lazear, Malmendier and Weber (2009). The authors offered subjects

In assuming that (some) investors distinguish between *ambiguity* (by which we mean uncertainty with unknown probabilities) and *risk* (by which we mean uncertainty with known probabilities) we follow in the tradition of Knight (1939) and Ellsberg (1961).<sup>2</sup> (Of course the familiar tradition that follows Savage (1954) assumes that investors behave ‘as if’ they assign complete subjective probabilities in every situation that involves uncertainty.) Ambiguity aversion, like risk aversion, is a characteristic of individuals, but it may arise in different ways. Heath and Tversky (1991) and Fox and Tversky (1995) find that many individuals prefer ambiguous bets when they feel especially knowledgeable about underlying events or especially capable of evaluating them, and prefer risky bets otherwise: ambiguity aversion may be driven by a “feeling of incompetence.” Fox and Tversky (1995) suggest that “people’s confidence is undermined when they contrast their limited knowledge about an event with their superior knowledge about another event, or when they compare themselves with more knowledgeable individuals.” They refer to this phenomenon as *comparative ignorance*.

Ambiguity aversion matters in our setting because the behavioral consequences of ambiguity aversion may be quite different from the behavioral consequences of risk aversion. In particular, ambiguity aversion may lead investors to avoid ambiguity altogether and to choose an portfolio whose payoffs are identical in the ambiguous states; in particular, such agents’ holdings of ambiguous securities will not be dependent on prices, at least for a wide range of prices. The pricing of ambiguous securities therefore will be (essentially) determined by the holdings of agents who do not perceive ambiguity or are not ambiguity averse, and whose choices *will be* dependent on prices. (Note that risk aversion will seldom if ever lead an investor to choose a riskless portfolio, so that the choices of investors who are simply risk averse *will be* reflected in prices.) This is precisely the experimental finding of Bossaerts, Ghirardato, Guarnaschelli and Zame (forthcoming).<sup>3</sup>

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the choice to play a dictator game or to opt out. They found that many subjects opted out and that behavior of the remaining subjects is skewed in comparison with behavior in experiments where subjects cannot opt out. The suggestion is that subjects who opt out have different attitudes toward selfishness/generosity but make choices that do not reveal/express those attitudes. Similarly, our results suggest that subjects who update incorrectly and perceive ambiguity make choices that do not reveal/express their updating errors.

<sup>2</sup>Some authors use *Knightian uncertainty* for what we call ambiguity.

<sup>3</sup>A *caveat* must be understood here. An agent who does not purchase a particular security does not *directly* affect the price of that security, but might *indirectly* affect the price because his/her holding of *other* securities affects supplies; hence the prices of *all* securities might be different from what they would be if that agent

Ambiguity – or at least the perception of ambiguity – arises in our setting because we give subjects information about investments in a way that presents them with complicated Bayesian updating problems. For instance, we offer two Arrow securities whose payments depend on whether a card drawn at the end of trading is red or black. Initially, the deck of cards contains one of each suit (spades, hearts, diamonds, clubs), so the prior probability of red/black is .5. Midway through the trading period we reveal one card that will *not* be drawn – but we *never* reveal a spade. (As the reader may surmise, our experimental design is inspired by the well-known ‘Monty Hall problem,’ but (many of the) updating problems we pose are (transparently) more complicated.<sup>4</sup>)

Because we pose complicated updating problems, we hypothesize that many investors lack confidence in their updating ability, and thus view themselves as comparatively ignorant. This lack of confidence may be reinforced if such investors are confronted with prices that appear at odds with their (incorrectly) updated beliefs. Such investors may treat security payoffs as ambiguous – rather than risky – and ambiguity aversion may lead these investors to choose an unambiguous portfolio independently of the prices of these securities (or at least for prices in a range that includes the observed prices). As discussed above, the choices of such *price-insensitive* investors do not contribute (directly) to the determination of the corresponding security prices.<sup>5</sup> In contrast, agents who are confident in their updating ability *know* (or behave as if they know) the true probabilities over outcomes, do not experience comparative ignorance, and regard security payoffs as risky – rather than ambiguous. Risk aversion affects the choices of such investors but does not lead them to choose riskless portfolios; hence the

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were entirely absent from the market. See Bossaerts, Ghirardato, Guarnaschelli and Zame (forthcoming) for an extended discussion.

<sup>4</sup>Monty Hall was the host of a popular weekly television show (aired in the 60’s) called “Let’s Make a Deal.” In one portion of the show, Monty would present the contestant with three doors, one of which concealed a prize. Monty would ask the contestant to pick a door; after which Monty – who *knew which door concealed the prize* – would open one of the two remaining doors, never revealing the prize. Monty would then offer the contestant the opportunity to switch to the other unopened door. Updating correctly demonstrates that the probability that the original door conceals the prize is 1/3 – as it was initially – so that switching dramatically increases the probability of success. However, many contestants – and others – update incorrectly and believe that the the probability that the original door conceals the prize is 1/2, so that switching makes no difference. For a detailed overview of the problem and its solution, see <http://mathforum.org/dr.math/faq/faq.monty.hall.html>.

<sup>5</sup>An alternative theory that would lead to the same individual behavior and would also stem from the comparative ignorance argument is that of Chew and Sagi (2008). A trader who doubts her Bayesian inference would prefer sources of uncertainty that do not depend on the Bayesian inference in question. The traders with such preferences for sources will choose portfolios that pay the same across all states with uncertain probabilities. The pricing implications of this individual behavior would be the same as under ambiguity aversion.

choices of such *price-sensitive* investors do contribute to the determination of the corresponding security prices. (Of course some subjects who are confident in their updating ability may be nevertheless be *wrong*; hence we do not expect prices to conform perfectly to theoretical predictions under the assumption that all investors update correctly.)

Thus, if subjects who have cognitive biases are led to perceive ambiguity when it is hard for them to solve difficult inference problems, the very cognitive biases that caused them to perceive ambiguity in the first place *will not* be (directly) reflected in prices. Instead, prices will be determined by those who do not perceive ambiguity because they *do not have* these cognitive biases.<sup>6</sup>

In order to obtain clear theoretical predictions about behavior and pricing in the absence of cognitive biases, we design our main experiment in such a way that there is no aggregate risk. In the absence of cognitive biases, therefore, risk-neutral pricing should obtain in equilibrium. That is, in the absence of cognitive biases, prices are predicted to be expectations of final payoffs, conditional on the information provided. The issue is, of course, whether these prices reflect expectations with respect to the true probabilities, or with respect to some other set of (biased) probabilities.<sup>7,8</sup> The absence of aggregate risk is important for understanding prices, as only in this case are predicted prices independent of risk attitudes of subjects (but assuming risk aversion); without this assumption it would not be possible to compare prices predicted by theory with prices observed in the experiments. However, the presence of aggregate risk provides a useful test of *individual* behavior, so in addition to the main experiment with no aggregate risk we conduct three sessions with aggregate risk.

Our central predictions are: i) subjects who cannot update correctly should hold ambiguity-neutral portfolios (in our setting, these correspond to balanced portfolios), and ii) the wedge

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<sup>6</sup>But note that attitudes toward risk might be correlated with cognitive abilities/biases.

<sup>7</sup>The presence of ambiguity aversion does not alter this conclusion, because ambiguity averse subjects are able to trade to risk-free positions (thereby avoiding exposure to probabilities they cannot compute) without generating aggregate risk for the remainder of the market. Hence, their demands do not create an imbalance in the risk available to subjects who do not perceive ambiguity, so equilibrium prices should continue to be expectations of final payoffs.

<sup>8</sup>The absence of aggregate risk also ensures that equilibrium (with strictly positive prices) exists even if *all* subjects are extremely ambiguity averse. In that case, prices will not be expectations of final payoffs. It can be shown that any price level would be an equilibrium, and that prices would be insensitive to the information provided.

between the portfolio composition of agents who can and cannot update correctly should grow with the aggregate risk in the economy. These predictions are fully born out in the data. In particular, our experimental data suggest that relatively few subjects solve the updating problems correctly and that many of these subjects treat the situation as ambiguous, rather than risky. We also find that price predictions are generally born out, but that the extent to which observed prices conform to theoretical predictions deteriorates significantly as the number of subjects who cannot make the correct Bayesian inferences increases.

We are not the first to experimentally investigate the effects of ambiguity and ambiguity aversion on asset prices. Bossaerts, Ghirardato, Guarnaschelli, and Zame (forthcoming) address the issue in the case of asset markets with both risky and ambiguous securities. While ambiguity is exogenous in their design, here we assume that the perception of ambiguity emerges endogenously. Our finding that agents who face uncertainty seek balanced positions is in line with what these authors find.

Our results shed light on recent experimental findings of Kluger and Wyatt (2004) who also used a design suggested by the Monty Hall problem. Kluger and Wyatt found that if at least two among the six subjects in an experimental market updated correctly, then prices agreed with theoretical predictions. They authors explain this finding as resulting from Bertrand competition among those who update correctly. It seems to us that this explanation begs the question: surely subjects who update *incorrectly* Bertrand compete as well?<sup>9</sup> And if subjects who update incorrectly Bertrand compete, why wouldn't this competition lead to the *wrong* prices? We provide an alternative explanation: those who cannot compute the right probabilities perceive ambiguity, and, as a result, become infra-marginal.

Others have studied the impact of cognitive biases on financial markets. Coval and Shumway (2005) document that loss aversion has an impact on intra-day price fluctuations on the Chicago Board of Trade, but only over very short horizons. Our study uses controlled experiments. We

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<sup>9</sup>Theoretically, irrational traders will be bankrupt in the long run but as shown by Kogan, Ross, Wang, and Westerfield (2006), they can survive for a very long time before their wealth is brought down to zero. Moreover, Kogan et. al show that if they act as expected utility maximizers under their subjective beliefs, "irrational traders can maintain a persistent influence on prices even after they have lost most of their wealth," (p. 220).

focus on pricing relative to theoretical levels. By virtue of experimental control, we know what the theoretical price levels are, unlike in field research such as Coval and Shumway (2005).

Our results also shed light on the relevance of experiments for finance. Our experiments provide a microcosm of field markets, and in particular, are populated with subjects who exhibit cognitive biases, but they are not an identical replica, because the pool of subjects from which we draw is not identical with the pool of investors in field markets. In fact, we find strong *cohort effects* in our experiments: the number of subjects who do update correctly, and hence the extent to which observed prices conform to theoretical predictions, depends strongly on the student pool from which our subjects are drawn. Because of this, our experiments provide little direct information about “mispricing” in field markets. The experiments are relevant for finance, though, because they provide a link between cognitive biases and equilibrium asset pricing – through the perception of ambiguity. Finally, our findings also suggest expanded role of financial markets, beyond risk sharing and information aggregation, to facilitating social cognition. That markets may facilitate social cognition was first suggested in Maciejovsky and Budescu (2005) and Meloso, Copic, and Bossaerts (2009).

The remainder of this paper is organized as follows. Section II presents the theory and the empirical implications. Section III describes our experiments in detail. Section IV presents the empirical results. Finally, Section V concludes.

## II. Theory and Empirical Implications

In this section we present a simple asset market model that unfolds over two dates: trade takes place only at date 0; consumption takes place only at date 1. There is a single consumption good.

Let there be a continuum of agents uniformly distributed on the interval  $[0, 1]$  and indexed by  $i$ , two assets  $R$  and  $B$ , or Red and Black stock, and two states of the world,  $r$  and  $b$ . At date 0 the realization of the state is not known to the agents. At date 1 agents learn the realization of the state, securities pay off, and consumption takes place. The two assets are

Arrow securities: In state  $j \in \{r, b\}$ , asset  $J \in \{R, B\}$  pays one unit of wealth, and the other asset pays no wealth.

Let  $\pi_r$  be the probability that state  $r$  occurs, and  $\pi_b = 1 - \pi_r$  the probability that state  $b$  occurs (note that  $\pi_j$  is equal to the expected value of asset  $J$ ). This probability is not common knowledge, but it is common knowledge that it can be computed using the publicly available information. Agents, however, may have cognitive biases that lead them to computational errors. Let  $\pi_j^i$  be the subjective probability that state  $j$  occurs, as calculated by agent  $i$ . We assume that a proportion  $\alpha$  of all agents can compute the correct probability. Specifically, we assume that  $\pi_r^i = \pi_r$  for  $i \in [1 - \alpha, 1]$ . The rest of the agents  $i \in [0, 1 - \alpha]$  compute the probability of state  $r$  incorrectly. In particular, agent  $i$  has a subjective probability  $\pi_r^i = \pi + \frac{\pi_r - \pi}{1 - \alpha}i$ . The beliefs are depicted in Figure 1. Note that we have chosen the true probability to be on the boundary of the belief space, thus creating a setup where the agents with wrong beliefs have the strongest potential to influence asset prices.<sup>10</sup>

We assume that the aggregate endowment in the economy of assets  $R$  and  $B$  is the same, so there is no aggregate risk in the economy. However, we do discuss the implications of the theory also for the setup with aggregate risk. At date 0 each agent is endowed with one unit of  $R$  and one unit of  $B$  (one can think of each agent as the aggregation of heterogeneously endowed agents who share the same beliefs  $\pi_r^i$ ). Let  $w_i$  be the wealth of agent  $i$  at date 1, after the state of the world is revealed. For simplicity, assume that  $u(w_i) = \ln(w_i)$  is the utility that agent  $i$  derives from final wealth.

Agents can trade their endowments at date 0. Let  $p_R$  be the market prices of asset  $R$  at date 0 (absence of arbitrage dictates that the price of asset  $B$  must be  $p_B = 1 - p_R$ ). Consider an agent  $i$  who maximizes expected utility according to her own subjective probabilities  $\pi_j^i$  and let  $(B_i, R_i)$  be her date 1 portfolio. The initial wealth of  $i$  is  $w_i^0 = p_R + (1 - p_R) = 1$ , so her optimization problem is

$$\max_{R_i, B_i} \pi_r^i \ln(R_i) + (1 - \pi_r^i) \ln(B_i) \quad s.t. \quad p_R R_i + (1 - p_R) B_i = 1.$$

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<sup>10</sup>Unless the true probability holds a knife-edge position in the beliefs interval, such that the price of Red stock happens to be always correct due to the symmetric distribution of beliefs around the correct probability, the comparative static conclusions of the theory would continue to hold.

The first order conditions for optimality imply that

$$R_i = \frac{\pi_r^i}{p_R}, \quad B_i = \frac{1 - \pi_r^i}{1 - p_R}$$

Hence for any given price vector, the relative demand of agent  $i$  for asset  $R$  (as a fraction of the total demand for assets  $R$  and  $B$ ) is increasing in the subjective probability  $\pi_r^i$ . The vector of all subjective probabilities by all agents determines the equilibrium prices.

If all agents correctly compute the true probability of state  $j$ , i.e., if  $\alpha=1$ , in the absence of aggregate uncertainty the equilibrium prices are  $p_R = \pi_r$  and  $p_B = 1 - \pi_r$  and in equilibrium all agents trade so as to attain a balanced portfolio.<sup>11</sup>

If, instead,  $\alpha < 1$  and if all agents maximize expected utility then the equilibrium prices will reflect the beliefs of all agents. The assumed beliefs distribution would imply that  $p_R < \pi_r$  and  $p_B > \pi_b$ . The equilibrium notion when beliefs are heterogeneous assumes that when confronted with prices that contradict their computations agents continue to use their subjective beliefs in determining optimal demands. In what follows we relax this very assumption. Instead, we assume that from the agents who do not hold the correct belief  $\pi_R$ , only those whose beliefs are  $\epsilon$ -close to the market price continue to use their subjective probabilities. Each of the rest of the agents, confronted with the divergence between the market price and her subjective probability, comes to realize that there must be agents, possibly herself, who have computed the wrong probabilities. We conjecture that in these circumstances the agents no longer trust their own computations, i.e., they experience *comparative ignorance*. As argued by Fox and Tversky (1995), comparative ignorance triggers ambiguity aversion. Thus, the agents who no longer trust their subjective probabilities become unsure about the true probabilities. As a result, they no longer face risk and instead face (Knightian) uncertainty in the marketplace.

For simplicity we assume that the agents who perceive ambiguity apply the maxmin decision rule (see Gilboa and Schmeidler (1989)).<sup>12</sup> While we present the details of the theory in the

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<sup>11</sup>In the case when there is aggregate risk and, say, the Red asset is more scarce than the Black asset, the equilibrium price ratio  $\frac{p_R}{p_B}$  is higher than  $\frac{\pi_r}{1-\pi_r}$ , with its exact level being determined by the scarcity of Red and the risk attitudes of the agents.

<sup>12</sup>Alternatively, one can use the theory in Ghirardato, Maccheroni, and Marinacci (2004) to model the behavior of agents who face ambiguity. The derived representation is  $\alpha - \max \min$  utility function  $U_i(R_i, B_i) = \alpha \min\{u(R_i), u(B_i)\} + (1 - \alpha) \max\{u(R_i), u(B_i)\}$ , where the coefficient  $\alpha$  measures the degree of ambiguity

Appendix, the main result is that agents who perceive ambiguity become price insensitive: they do not adjust their portfolios in response to changes in prices, seeking a balanced portfolio regardless of price fluctuations. Hence, when there is no aggregate risk, ambiguity averse agents do not affect prices, and prices are set by those agents who stick to their subjective probabilities. If there is aggregate risk, ambiguity averse agents who seek a balanced portfolio would drive up the price of the relatively scarce asset. In addition, the agents who do not experience comparative ignorance would end up holding the risky part of the aggregate portfolio in equilibrium. Thus, in the aggregate risk case there is a greater wedge between the risk composition of the portfolios of the risk-averse and the ambiguity-averse agents.

Formally, we make the following key assumption.

**Assumption CI (Comparative Ignorance)** *Agents who compute the correct probabilities use them in computing optimal demands independent of the price levels. Agents who compute wrong probabilities feel comparative ignorance if the price ( $p_R$  or  $p_B$ ) is more than  $\epsilon$  away from their subjective probability ( $\pi_r^i$  or  $\pi_b^i$ ) and adopt max min preferences. Agents with wrong subjective probabilities that are within  $\epsilon$  of the prevailing prices use their subjective probabilities when computing optimal demands.*

To reiterate, this assumption means that people who are right (in computing the underlying probabilities) are certain and are not swayed in their certainty when prices diverge from the theoretical prediction, whereas people who cannot compute probabilities are not as certain of their calculations and they lose their confidence as soon as market prices do not correspond to the prices that should occur in equilibrium given the calculated probabilities. Providing support to our assumption, Halevy (2007) finds that 95% of agents who fail at the standard calculation task of reducing compound lotteries avoid ambiguity in the Ellsberg experiment, whereas only 4% of agents who correctly reduce compound lotteries exhibit this behavior.

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aversion.  $\alpha = 1/2$  corresponds to ambiguity neutrality, and  $\alpha = 1$  is the extreme degree of ambiguity aversion as in Gilboa and Schmeidler (1989). BGGZ use the  $\alpha$  – max min for their theoretical analysis.

**Definition** An economy  $\mathcal{E}$  with Arrow assets  $R$  and  $B$  consists of a family of agents with mass 1 distributed on  $I = [0, 1]$ , each with utility function of wealth  $u(w) = \ln(w)$ , and initial endowments  $(R_i^0, B_i^0) = (1, 1)$ . The following assumptions are always in force.

- (a) Assumption CI.
- (b)  $\epsilon < \frac{\alpha}{1+\alpha}D$ , where  $D = \pi_r - \underline{\pi}$ .

As provided in the Appendix, assumption (b) ensures that a strictly positive fraction of agents become price-insensitive in equilibrium through the channel of comparative ignorance. If no agent becomes price-insensitive, the resulting equilibrium is one where all agents are expected utility maximizers although with heterogeneous beliefs. Assumption (b) is also a sufficient condition (again shown in the Appendix) for the conformation of observed prices with theoretical predictions to improve with the number of price sensitive agents,  $S$ . In other words, it guarantees that the distance between the true probability  $\pi_r$  and the price  $p_R$  be a decreasing function of  $S$ . Notice that the group of price-sensitive agents is comprised by the agents who can compute the correct probabilities in addition to the agents who have an error in computing the probabilities, however they are  $\epsilon$ -close to it. As shown in the Appendix, the distance between  $\pi_r$  and  $p_R$  is always decreasing in  $\alpha$ . However, in the experiment the proportion  $\alpha$  is measured with greater noise than the proportion of the price-sensitive agents  $S$ . Assumption (b) allows us to use the estimate with greater precision,  $S$ , in addition to the estimate of  $\alpha$  for our empirical tests.

**Proposition 1.** *The equilibrium price of the Red stock in the economy  $\mathcal{E}$  is*

$$p_R = \pi_r - \left( \sqrt{\left(\frac{\alpha D}{1-\alpha}\right)^2 + \epsilon^2} - \frac{\alpha D}{1-\alpha} \right)$$

**Corollary 1.** *The extent to which observed prices conform to theoretical predictions increases in the fraction of agents  $S = \alpha + (\epsilon + \pi_r - p_R) \frac{1-\alpha}{D}$  who do not experience comparative ignorance.*

**Corollary 2.** *The extent to which observed prices conform to theoretical predictions, as measured by  $-(\pi_r - p_R)$  increases in  $\alpha$ , the fraction of agents who can derive the correct probabilities and decreases in  $\epsilon$ .*

Thus, our theory has three testable empirical predictions:

**Hypothesis 1.** *The number of price-sensitive subjects,  $S$ , is positively related with the extent to which observed prices conform to theoretical predictions in the experimental markets – the higher the number of price-sensitive subjects, the smaller the mispricing  $(|\pi_r - p_R|)$ .*

**Hypothesis 2.** *The number of agents who can correctly compute the probability  $\pi_R$  is positively related with the extent to which observed prices conform to theoretical predictions in the experimental markets, i.e., it is negatively related with the mispricing  $(|\pi_r - p_R|)$ .*

**Hypothesis 3.** *According to our assumption (CI), given  $|\pi_r - p_R| > 0$ , price insensitive subjects hold more balanced portfolios than price insensitive subjects. The effect is stronger in setups where there the aggregate endowment is risky.*

### III. Experiments

The experimental sessions were organized as a sequence of *independent* replications, referred to as *periods*, of four different situations and where each situation was repeated exactly twice.

Twenty subjects participated in each session. This is sufficient for markets to be liquid enough that the bid-ask spread is at most two or three ticks (the tick size was set at 1 U.S. cent). All accounting in the experiments was done in US dollars. The average earnings from participating in the experimental sessions was \$49 per subject.

There were six sessions for the main experiment. The sessions were ran at the following universities: (i) Caltech (one session), (ii) UCLA (one session), (iii) University of Utah (two sessions), (iv) simultaneously at Caltech and University of Utah with equal participation from both subject pools (two sessions).

There were three securities in the laboratory markets, two of them were risky and one was risk free. Trade took place through a web-based, electronic continuous open-book system called *jMarkets*.<sup>13</sup> A snap shot of the trading screen is provided in Figure 2.

The (two) risky securities were referred to as *Red Stock* and *Black Stock*. The liquidation value of Red Stock and Black Stock was either \$0.50 or \$0. Red and Black Stock were complementary securities: when Red Stock paid \$0.50, Black Stock paid nothing, and *vice versa*. Red Stock paid \$0.50 when the “last card” (to be specified below) in a simple card game was red (hearts or diamonds); Black Stock paid \$0.50 when this “last card” was black (spades or clubs).

Subjects were initially endowed with both Red and Black Stock. They were allowed to trade Red Stock, but *not* Black Stock. This is an important experimental design feature, as in an environment with no aggregate risk, where there is a risk free means of trading (cash), and where all securities can be traded, the equilibrium allocations are indeterminate. Fixing allocation of Black Stock in the agents’ portfolios and barring trade in Black Stock, provides unique equilibrium allocation predictions for Red Stock.

In addition, since subjects were initially given an unequal supply of the two securities, and given small but significant risk aversion that is known to emerge for the amount of risk we induced in our experiments (see Holt and Laury (2002)), there was a reason to trade even if all subjects were able to compute the correct probabilities.

Subjects could also trade a risk free security called *Note*. This security always paid \$0.50. Because of the presence of cash, the Note was a redundant security. However, subjects were allowed to short sell the Note if they wished. Short sales of Notes correspond to borrowing. Subjects could exploit such short sales to acquire Red Stock if they thought Red Stock was underpriced.

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<sup>13</sup>This open-source trading platform was developed at Caltech and is freely available under the GNU license. See <http://jmarkets.ssel.caltech.edu/>. The trading interface is simple and intuitive. It avoids jargon such as “book,” “bid,” “ask,” etc. The entire trading process is point-and-click. That is, subjects do not enter numbers (quantities, prices); instead, they merely point and click to submit orders, to trade, or to cancel orders.

Subjects were also allowed to short sell Red Stock, for in case they thought Red Stock was overpriced. To avoid bankruptcy (and in accordance with classical general equilibrium theory), our trading software constantly checked subjects' budget constraints. In particular, subjects could not submit an order such that, if it and the subject's other standing orders were to go through, the subject would generate net negative earnings in at least one state. Only new and standing orders that were within 20% of the best standing bid or ask in the marketplace were taken into account for the bankruptcy checks. Since markets were invariably thick, orders outside this 20% band were effectively non-executable, and hence, deemed irrelevant. The bankruptcy checks were effective: no agent ever ended up with negative earnings in our experiments.

In all sessions of the main experiment the aggregate endowments of Red and Black stock were equal. In addition, three sessions were conducted within a setup with aggregate risk. The only difference between these sessions and the ones of the main experiment is that there were more subjects with endowments tilted towards Red stock than subjects with endowments tilted towards Black stock. All sessions with aggregate risk were ran at UCLA. As discussed earlier, one major problem with analyzing the pricing data from those experiments is that the experimenter does not have the theoretical prediction of the equilibrium prices when all agents have the correct probabilities, because the experimenter has no control over the aggregate risk aversion of the participants. However, the setup with aggregate risk provides a stronger platform for testing our crucial assumption (CI).

Table I provides details of the experimental design. Note the \$5 sign-up reward, compulsory at the experimental laboratories where we ran our experiments (Caltech's SSEL, UCLA's CASSEL and the University of Utah's UULEEF). The sign-up reward was for subjects to keep no matter what happened in the experiment. Hence, it constituted the minimum payoff (for an experiment that generally lasted 2 hours in total).<sup>14</sup>

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<sup>14</sup>The instructions for the experiment are provided in the Appendix. More information about the experimental design can be obtained at [http://leef.business.utah.edu/market\\_mh/frames\\_mh.html](http://leef.business.utah.edu/market_mh/frames_mh.html).

The liquidation values of Red and Black Stock were determined through card games played by a computer and communicated to the subjects orally and through the News web page. The card games were inspired by the Monty Hall problem.

One game (out of the four that we used) is as follows. The computer starts a new period with four cards (one spades, one clubs, one diamonds, and one hearts), randomly shuffled, and face down. The computer discards one card, so there are three remaining cards. The color of the “last card” determines the payoffs of the two risky securities. Trade starts. Halfway through the period, trading is halted temporarily. During the trading halt the computer picks one card from the remaining cards as follows. If the discarded card was hearts, the computer picks one card at random from the three remaining cards. If the hearts is in the three remaining cards, the computer picks randomly from the other two (non-heart) cards. The card that was picked is then revealed to the subjects, both orally and through the News web page. Trade starts again. At the end of the period, after markets close, the computer picks one of the two remaining cards at random. This *last card* is then revealed and determines which stock pays. If the last card is red (diamonds, hearts) then Red Stock pays \$0.50. If the last card is black, then Black Stock pays \$0.50.

Detailed information about the drawing of cards is in the set of experimental instructions provided to the subjects. In addition, before each period, the experimenter reiterated the drawing rules to be applied in the coming period.

Four variations on this game (each replicated twice), referred to as *treatments*, were played. They differ in terms of the number of cards initially discarded, the number of cards revealed mid-period, and the restriction on which cards would be revealed. This provided a rich set of equilibrium prices and changes of prices (or absence thereof) after mid-period revelation. Table II provides details of the four treatments.

The actual trading within the eight periods lasted about one hour. It was preceded by a long (approximately one hour) instructional period and a practice trading session, followed by a short break (15 minutes). The purpose of the long instructional period and the trading practice session was to familiarize subjects with the setting and the trading platform. To determine to

what extent subjects understood the instructions, the (oral) questionnaire included questions such as “In the game where the computer never reveals a red card halfway in the period, will you be surprised to see a black card revealed?” Or, “If the computer initially discards one card, and then shows one black card when it could have also shown diamonds, does the chance that the last card is black decrease as a result?” Subjects were never told the correct probability levels, however.

## IV. Empirical Analysis

Given the experimental design, and with the conjectures about the impact of cognitive biases on ambiguity perception and its effect on equilibration and equilibrium in financial markets in mind (developed in Section II), we now refine our hypotheses.

The first goal of the study is to determine, for each experimental session, whether there are price-insensitive (infra-marginal) subjects and how their number affects the extent to which observed prices conform to theoretical predictions .

**Hypothesis 1.** *The number of price-sensitive subjects,  $S$ , is positively related with the extent to which observed prices conform to theoretical predictions in the experimental markets – the higher the number of price-sensitive subjects, the smaller the mispricing ( $|\pi_r - p_R|$ ).*

The monotonicity of  $|\pi_r - p_R|$  with respect to the number of agents who can compute the correct probability (a result that holds for all values of the parameters  $\alpha$  and  $\epsilon$ ) implies our second hypothesis.

**Hypothesis 2.** *The number of agents who can correctly compute the probability  $\pi_R$  is positively related with the extent to which observed prices conform to theoretical predictions in the experimental markets, i.e., it is negatively related with the mispricing ( $|\pi_r - p_R|$ ).*

The third hypotheses concerns differences in allocations between price-sensitive and price-insensitive agents, as postulated in assumption CI behind our theory.

**Hypothesis 3.** *According to assumption CI, given  $|\pi_r - p_R| > 0$ , price insensitive subjects hold more balanced portfolios than price sensitive subjects. The effect is stronger in setups where there the aggregate endowment is risky.*

The ambiguity averse (price-insensitive) agents prefer to hold balanced portfolios irrespective of prices. In the sessions with no aggregate risk, this hypothesis would hold only when the price  $p_R$  is not equal to the probability  $\pi_r$ . Indeed, when the price of Red stock is equal to its expected payoff even price-sensitive subjects seek balanced positions (provided they are risk averse). Thus, in our empirical analysis we control for the level of mispricing when testing the third hypothesis. In the setup with aggregate risk, the price-insensitive agents should hold more balanced portfolio independent of the level of mispricing.

In what follows we describe the data, assess the level of mispricing in each treatment for the six main sessions, and present the procedure for determining the price sensitivity of subjects along with estimates of their numbers. We then proceed to test the three main hypotheses of our study.

## A. Experimental Data

The data collected during the experiments consists of all posted orders and cancelations for all subjects along with their transactions and the transaction prices for the Red Stock and the Note. Figure 3 displays the evolution of transaction prices for Red Stock in the six experimental sessions. Time is on the horizontal axis (in seconds). Solid vertical lines delineate periods; dashed vertical lines indicate half-period pauses when the computer revealed one or two cards. Horizontal line segments indicate predicted price levels assuming prices equal expected payoffs computed with correct probabilities. Each star is a trade. Over 1,100 trades take place typically, or one transaction per 2.5 seconds.

Figures 3b and 3c display trading prices in experiments that represent two extremes. Indeed, observed prices conform *very badly* with theoretical predictions in the University of Utah-1 experiment (Figure 3b). However, when Caltech students are brought in (Figure 3c, where half of the subjects are from Caltech, and half are from the University of Utah), prices

are close to expected payoffs – the conformity with theoretical predictions overall is *good*. The comparison suggests that there might be strong cohort effects in our data.

In the University of Utah experiment (Figure 3b), prices appear to be insensitive to the treatments. There were also a large number of price-insensitive subjects (to be discussed later), suggesting that the pricing we observe in that experiment may reflect an equilibrium with only ambiguity averse subjects: when there are only ambiguity averse subjects, equilibrium prices will not react to the information provided to subjects in different treatments, and *any* price level is an equilibrium. Notice that prices in the University of Utah experiment indeed started out around the relatively arbitrary level of \$0.45 and stayed there during the entire experiment. A notable exception is the second half of period 1, when it was certain that the last card would be red and hence that the Red Stock would pay, because the two revealed cards were black. Prices adjusted correctly, proving that subjects were paying attention and able to enter orders correctly, so that neither lack of understanding of the rules of the game or unfamiliarity with the trading interface can explain the treatment-insensitive pricing in the other periods.

Column I of table III reports conformity of observed prices with theoretical predictions for all treatments in each experiment. Conformity is measured in terms of mean absolute mispricing (in U.S. cents) across transactions. As in section II, let  $\pi_r$  denote the true probability that Red stock pays. Because payoff-relevant information is revealed in the middle of each trading period, this true probability takes on two values,  $\pi_{r1}$  in the first half of the trading session, and  $\pi_{r2}$  in the second half. For each transaction, the absolute mispricing is computed as the absolute difference between the transaction price and the corresponding value of  $\pi_r$ . Column I reveals that there is a wide variability in mispricing, both across experiments, Utah producing the worst mispricing and Caltech-Utah producing the best pricing, and across treatments, with treatment 2 producing larger mispricing than the other treatments. Formally, the median mispricing in treatment 2 is significantly higher than that of treatment 1 ( $p$  value of 0.047 on the Wilcoxon signed-rank test comparing the paired absolute mean mispricing across the two treatments), treatment 3 ( $p = 0.016$ ), and treatment 4 ( $p = 0.016$ ).

For completeness, we also report the mean absolute differences between transaction prices and true probabilities for the sessions with aggregate risk. As argued earlier, however, the

reported numbers cannot immediately be interpreted as mispricing because the correct price is no longer equal to the expected payoff of the Red Stock.

## B. Empirical Tests

Can we explain the variability in mispricing in terms of the number of price-sensitive subjects, as we conjectured (Hypothesis 1)? Column II of table III reports the number of price-sensitive subjects. Price sensitivity is obtained from OLS projections of the one-minute changes in a subject's holdings of Red Stock onto the difference between, (i) the mean traded price of Red Stock (during the one-minute interval), and (ii) the expected payoff of Red Stock computed using the correct probabilities. Agents who do not become ambiguity averse are expected to be price sensitive and should be detectable through this regression because their choices generate a negative slope coefficient. In contrast, ambiguity averse agents become price insensitive, which means that their choices generate a zero slope coefficient in the above regression.

As we argue in the Appendix, however, the requirement that total changes in holdings balance out causes a well-known simultaneous-equation effect, which biases the slope coefficients upward (the slope coefficients of all subjects mechanically sum up to zero). Because coefficients are upward biased, we decided to use a generous cut-off level to classify our subjects. We chose a cut-off of -1.65 for the  $t$ -statistic of the slope coefficient to indicate that the subject is price sensitive because she tends to reduce holdings when prices increase. At the same time, we used a conservative  $t$ -statistic level of 1.9 to determine whether a subject is price sensitive in the other direction, namely, she increases holdings of a security when prices increase (rather perversely, as we shall discuss later). Conversely, subjects with  $t$ -statistics between -1.65 and 1.9 are classified as price-insensitive.

Table III demonstrates that the number of price-sensitive subjects was often very low. The flip side of this is that often many subjects were price-insensitive; their actions did not depend on prices. In some instances only *a single* or even *no* subject was found to react systematically to price changes. This means that generally a large number of our subjects perceived ambiguity – suggesting that they did not know how to compute the right probabilities.

We do observe that a small fraction of subjects were price-sensitive in a perverse way: they tended to *increase* their holdings for increasing prices. The number of such subjects is reported in column III. There are two possible explanations of this finding. First, these are just type II errors: the subjects at hand are really price insensitive, but sampling error causes the  $t$ -statistic to be above 1.9. The second possibility is that we have identified subjects who are indeed perversely price sensitive. We could interpret their actions as reflecting *momentum trading* or *herding*: higher prices are interpreted as signaling higher higher expected payoffs (or future prices). Our theory does not account for such trading behavior. Since we cannot determine which of the two possible explanations applies, we exclude subjects with  $t$ -statistics above 1.9 from the remainder of our analysis. In a previous version of the paper (available on SSRN and upon request from the authors) we did present our analysis in two parts: one where we include the entire subject pool and one where we exclude those with significantly positive slope coefficients. None of our qualitative conclusions are affected by the exclusion of perversely price-sensitive subjects.

### **Hypothesis 1**

Table III indicates that pricing improves significantly (mean absolute mispricing is lower) when there are more price-sensitive subjects who reduce their holdings with increases in prices relative to the correct value ( $S$  in our theory). That is, conformity of observed prices with theoretical predictions and number of marginal subjects are significantly negatively correlated. The correlation is equal to -0.53 (with standard error of 0.146), in line with Hypothesis 1 and thus with our conjecture that comparative ignorance affects behavior when prices are not in line with ignorant subjects' beliefs.

### **Hypothesis 2**

We test Hypothesis 2 by fine-tuning our subject classification. Price-sensitive subjects should include those who are ignorant but whose beliefs are close enough to observed prices for them not to become ambiguity averse. We attempted to differentiate those subjects from the ones

that do correctly compute probabilities, as follows. In our regressions of holding changes onto differences of prices from (correct) expected payoffs, the signs of the intercepts for subjects who know how to compute probabilities correctly should correspond to the signs of the difference in initial holdings of Black and Red Stock. E.g., for a subjects with initial allotment of 3 units of Black Stock and 12 units of Red Stock, the intercept should be negative, reflecting that, when prices equal (correct) expected payoffs, this subject is reducing holdings of Red stock, to offset risk of her holdings of (nontraded) Black Stock.

Column IV of Table III reports the number of price-sensitive subjects for whom the sign of the intercept the price sensitivity regressions corresponds to the sign of the difference in holdings of Black and Red Stock. These should be price-sensitive subjects who can compute probabilities correctly, namely,  $\alpha$  in the theory. The correlation between mispricing and this adjusted number of price-sensitive subjects increases in magnitude, to -0.58, with a decreased standard error of 0.088. As expected, the relationship between mispricing and our (noisy) proxy for  $\alpha$ , is stronger (the magnitude of the coefficient is larger and the associated standard error is smaller) than the relationship between mispricing and  $S$ , the number of price-sensitive agents.

### **Hypothesis 3**

One alternative explanation for the support of Hypotheses 1 and 2 is that those who do not react to price changes are simply noise traders, and hence, not necessarily ambiguity averse. The more noise traders in the market, the worse the conformity of observed prices with theoretical predictions. If not rejected, Hypotheses 3 would speak against this explanation.

To test Hypothesis 3, we investigate the difference in individual imbalances (equal to the absolute difference between the units of Red and Black Stock in each subject's portfolio) between price-sensitive and price-insensitive subjects. If indeed the latter were noise traders, we should not expect to see any difference between the imbalances of those two groups. If, on the other hand, price insensitivity indicates perception of ambiguity, price-insensitive subjects

should aim at achieving balanced positions. Price-insensitive subjects would display lower imbalance than price-sensitive ones.

We compute individual imbalances at mid-period and at the end of the period. The portfolio imbalance of a subject is the absolute difference between the number of Red Stock and Black Stock she is holding. The imbalance analysis includes all experimental sessions as the general prediction that ambiguity averse subjects hold more balanced portfolios obtains independent of the aggregate risk in the market. However, the difference in the portfolio compositions of the price-sensitive and price-insensitive subjects should be larger when there is risk in the economy, because price-sensitive subjects as a group have to absorb the aggregate risk, i.e., the aggregate imbalance in number of Red and Black Stock in the economy. We therefore present the results from the individual imbalance analysis in three parts. Panel A of Table IV presents the results from the sessions with no aggregate risk. Panel B results include sessions with aggregate risk only. Finally, Panel C presents the analysis with all sessions included.

To reduce the impact from noise in the estimation of price sensitivity for a given subject, we implement the following two-level analysis:

$$\begin{aligned}
 I_i &= a + b_{between}T_i + \epsilon_i, \text{ where} \\
 I_{ij} &= I_i + \eta_{ij}, \\
 T_{ij} &= T_i + \xi_{ij}, \text{ and} \\
 \eta_{ij} &= b_{within}\xi_{ij} + \delta_{ij},
 \end{aligned}$$

where  $T_{ij}$  denotes subject  $i$ 's  $t$ -statistic of the slope coefficient in the price-sensitivity regression for treatment  $j$  and  $\delta_{ij}$ ,  $\xi_{ij}$ ,  $\eta_{ij}$ , and  $\epsilon_i$  are normally distributed random errors. We are interested in  $b_{between}$ , which provides a filtered estimate of the relationship between portfolio imbalance of a subject and her price sensitivity across treatments. We do not report the within-level parameter estimates ( $b_{within}$ ): although with the correct (negative) signs, none were ever significantly different from zero. As robustness check, we repeated our estimation with  $b_{within}$  fixed at 0; the results remained largely unchanged. Throughout, we used robust maximum

likelihood estimation. The first row of Table IV presents the results on the between-level estimates.

In six sessions, there is no aggregate uncertainty, so, provided prices correctly reflect expectations, all (risk averse) subjects should hold balanced portfolios. The second row of Table IV, therefore, presents a specification that factors the level of mispricing into the relation between price-sensitivity and portfolio imbalance, as follows:

$$\begin{aligned}
 I_i &= a + b_{between}MT_i + \epsilon_i, \text{ where} \\
 I_{ij} &= I_i + \eta_{ij}, \text{ and} \\
 MT_{ij} &= MT_i + \xi_{ij}, \text{ and} \\
 \eta_{ij} &= b_{within}\xi_{ij} + \delta_{ij},
 \end{aligned}$$

where  $M_{ij}$  is the mean absolute mispricing in treatment  $j$  of the session in which subject  $i$  participated.

When pricing is incorrect, smart subjects should hold imbalanced positions even in the sessions with aggregate uncertainty, so the relationship between price sensitivity and imbalance should be stronger. Conformity of observed prices with predictions is lower at mid-period, because the task of computing the expected payoff of Red Stock is harder before revelation of information. We therefore also report in Table IV results with imbalances and measures of mispricing computed only for data from the first half of the periods (before intermediate information is revealed).

Overall, Table IV confirms our conjecture that price-sensitive subjects tend to hold more imbalanced positions both at the middle and at the end of the period. As expected, the relationship between price sensitivity and imbalance is stronger (as measured by the  $t$  statistic of the slope coefficient or by the regression  $R^2$ ) at the middle of the period, when subjects' inference problem is harder. Further confirming our expectations, the results are strongest for sessions with aggregate risk, but they remain significant when all sessions are included in the analysis. However, if only sessions with no aggregate risk are included, one cannot

reject the null that there is no difference in imbalance among the subjects with different levels of sensitivity, although all slopes are negative both in the mid-period and end-of-period regressions.

Overall, the data therefore provide strong evidence for the conjecture that price-insensitive agents behave in an ambiguity averse manner, and against the alternative that price-insensitivity merely reflects noise trading.

Our last test studies an implication that does not immediately emerge from our (equilibrium) theory, but that can reasonably be expected to obtain in an *equilibration* version of our model. The test concerns how the total number of trades (volume) relates to the classification of subjects by price sensitivity. If price-insensitive subjects were indeed noise traders, they could reasonably be expected to trade more than price-sensitive ones. In contrast, a model of equilibration with the same agents as in our (equilibrium) theory should predict the opposite: agents who know the probabilities will keep on trading as long as the price moves, while price-insensitive agents (ambiguity averse agents) stop trading once their portfolios are balanced. Table V provides results from regressions similar to those provided in Equations (1) and (2) whereby imbalance is replaced with the number of trades as dependent variable. As shown in the table, the price-sensitive subjects indeed tend to trade more, but the evidence is rather weak. While the signs of the coefficients (with the exception of two) carry the expected negative signs, none of them is statistically significant. Albeit weak, the last test does provide additional support for our conjecture that price-insensitivity proxies for ambiguity-aversion and not for noise trading.

## **Robustness Checks**

We repeated our analysis using ordinary least square (OLS) regressions. The results for both individual imbalance regressions and trading volume regressions are reported in Table VI. Any difference of the results in this table relative to those in Tables IV and V would stem from the “within” level noise in our data. As evident from the table, while none of the qualitative conclusions change, all coefficients decrease in magnitude, sometimes significantly.

## V. Conclusions

Our experimental results demonstrate that often only a minority of subjects are price-sensitive (marginal). The number of price-insensitive (infra-marginal) subjects in each of the sessions and the four different situations within a session significantly impacts conformity of observed prices with predictions. With only a few of the price-sensitive subjects present, market prices remain closer to their starting point than to their equilibrium levels. We find that the infra-marginal agents hold more balanced portfolios than the marginal agents. These findings support our interpretation that some agents who are cognitively biased experience comparative ignorance when they observe market prices, so that they become ambiguity averse and seek a balanced portfolio, thus showing no sensitivity to prices. These agents and their biases, then, do not affect prices.

It has been suggested before that inability to perform difficult computations may translate into ambiguity aversion, but only in the presence of clear evidence that others may be better (see Fox and Tversky (1995)). It is particularly striking that financial markets exude the very authority that is necessary to convince subjects who cannot do the computations correctly that they really cannot, and hence, to perceive ambiguity. As such, the role of financial markets includes not only risk sharing and information aggregation, but extends to social cognition.

Our findings raise an important issue: what cognitive biases translate into ambiguity perception when played out in the context of financial markets? The issue is important, because, as theory predicts and our experiments confirm, ambiguity may keep prices from being affected by the cognitive biases that generated it.

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# Appendix

## A. Mathematical Details

An agent with max min preferences maximizes the following expression:

$$U_i(R_i, B_i) = \min\{u(R_i), u(B_i)\}$$

If  $R_i > B_i$ , then  $U_i(R_i, B_i) = u(B_i)$ . Similarly, if  $R_i < B_i$ , then  $U_i(R_i, B_i) = u(R_i)$ . From here it immediately follows that an agents with maxmin preferences will seek a portfolio with  $R_i = B_i$  under any prices  $p_R$  and  $p_B = 1 - p_R$ .

Let the price of R be  $p_R$ . An expected utility maximizing agent  $i$  with belief  $\pi_r^i$  maximizes

$$U_i(R_i, B_i) = \pi_r^i u(R_i) + (1 - \pi_r^i) u(B_i).$$

The solution to this agent's optimization problem given her endowment, which by assumption is one unit of each asset, is  $R_i = \frac{\pi_r^i}{p_R}$ .

### A.1. Excess demand of knowledgeable agents

Let  $q_\alpha$  denote the aggregate demand of red asset by the fraction  $\alpha$  of agents who are able to calculate the correct probabilities. Then  $q_\alpha = \int_{1-\alpha}^1 R_i = \alpha \frac{\pi_r}{p_R}$ . Thus, for any  $p_R < \pi_r$  the knowledgeable agents create excess demand  $\alpha(\frac{\pi_r}{p_R} - 1)$ .

### A.2. Excess demand of ambiguity averse agents

For any price  $p_R$  the agents demand risk-neutral portfolio. Because of the assumption of no aggregate endowment uncertainty for any subinterval of agents, the ambiguity averse agents create excess demand of 0.

### A.3. Excess demand of price sensitive biased agents

Note that from the assumption that  $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$  and  $\alpha \leq 1$ , it follows  $\epsilon < \frac{\pi_r - \underline{\pi}}{2}$ . Conjecture that  $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$ . Let  $\underline{i}$  be the agent such that  $\pi_r^{\underline{i}} = p_R - \epsilon$ , that is,  $\underline{i} = \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \epsilon - \underline{\pi})$ . The excess demand generated by the biased agents is

$$\int_{\underline{i}}^{1-\alpha} \left( \frac{\pi_r^i}{p} - 1 \right) di.$$

Since  $\pi_r^i = \underline{\pi} + i \frac{\pi_r - \underline{\pi}}{(1-\alpha)}$ ,

$$\begin{aligned} \int_{\underline{i}}^{1-\alpha} \left( \frac{\pi_r^i}{p} - 1 \right) di &= \int_{\underline{i}}^{1-\alpha} \left( \frac{\underline{\pi}}{p_R} - 1 \right) di + \int_{\underline{i}}^{1-\alpha} \frac{\pi_r - \underline{\pi}}{(1-\alpha)p_R} i di = -\frac{p_R - \underline{\pi}}{p_R} \int_{\underline{i}}^{1-\alpha} 1 di + \frac{\pi_r - \underline{\pi}}{(1-\alpha)p_R} \int_{\underline{i}}^{1-\alpha} i di = \\ &= \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} i^2 \Big|_{\underline{i}}^{1-\alpha} - \frac{p_R - \underline{\pi}}{p_R} i \Big|_{\underline{i}}^{1-\alpha} = \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} ((1-\alpha)^2 - \underline{i}^2) - \frac{p_R - \underline{\pi}}{p_R} (1-\alpha - \underline{i}) = \\ &= \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} (1-\alpha - \underline{i})(1-\alpha + \underline{i}) - \frac{p_R - \underline{\pi}}{p_R} (1-\alpha - \underline{i}) = \\ &= \frac{\pi_r - \underline{\pi}}{2(1-\alpha)p_R} \left( 1-\alpha - \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) \left( 1-\alpha + \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) - \frac{p_R - \underline{\pi}}{p_R} \left( 1-\alpha - \frac{1-\alpha}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) = \\ &= (1-\alpha) \frac{\pi_r - \underline{\pi}}{2p_R} \left( 1 - \frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) \left( 1 + \frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) - (1-\alpha) \frac{p_R - \underline{\pi}}{p_R} \left( 1 - \frac{1}{\pi_r - \underline{\pi}}(p_R - \underline{\pi} - \epsilon) \right) = \\ &= (1-\alpha) \frac{1}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - 2\underline{\pi} + p_R - \epsilon) - (1-\alpha) \frac{p_R - \underline{\pi}}{p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon) = \\ &= \frac{1-\alpha}{p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon) \left( \frac{1}{2}(\pi_r - 2\underline{\pi} + p_R - \epsilon) - (p_R - \underline{\pi}) \right) = \\ &= \frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon). \end{aligned}$$

Because  $(\pi_r - p_R - \epsilon) < 0$ , the excess demand is negative, i.e., the biased agents provide excess supply to the market.

### A.4. Equilibrium

In equilibrium the aggregate excess demand must be zero.

$$\frac{1-\alpha}{2p_R(\pi_r - \underline{\pi})} (\pi_r - p_R + \epsilon)(\pi_r - p_R - \epsilon) + \alpha \left( \frac{\pi_r}{p_R} - 1 \right) = 0 \Leftrightarrow$$

$$\begin{aligned}\frac{1-\alpha}{2p_R(\pi_r-\underline{\pi})}(\pi_r-p_R+\epsilon)(p_R+\epsilon-\pi_r) &= \alpha\left(\frac{\pi_r}{p_R}-1\right) \Leftrightarrow \\ \frac{1-\alpha}{2(\pi_r-\underline{\pi})}(\pi_r-p_R+\epsilon)(p_R+\epsilon-\pi_r) &= \alpha(\pi_r-p_R) \Leftrightarrow\end{aligned}$$

Denote  $\pi_r - p_R$  by  $y$ . Then

$$\frac{1-\alpha}{2(\pi_r-\underline{\pi})}(y+\epsilon)(\epsilon-y) = \alpha y$$

Denote  $\frac{\alpha}{1-\alpha}(\pi_r-\underline{\pi})$  by  $K$ . Then

$$y^2 + 2Ky - \epsilon^2 = 0$$

The (positive) solution to the equation is  $y = \sqrt{K^2 + \epsilon^2} - K$ . Note that  $\lim_{\epsilon \rightarrow 0} y = 0$ , i.e. the price converges to  $\pi_r$  as  $\epsilon$  converges to zero.

The above derived equilibrium satisfies the conjecture that  $p_R + \epsilon > \pi_r > p_R > p_R - \epsilon > \underline{\pi}$  as depicted in Figure 4.

### A.5. Comparative Statics

Since  $\frac{\partial K}{\partial \alpha} = \frac{(\pi_r - \underline{\pi})}{(1-\alpha)^2} > 0$  and  $\frac{\partial K}{\partial \alpha} \frac{\partial y}{\partial K} = \frac{K}{\sqrt{K^2 + \epsilon^2}} - 1 < 0$ , it follows that  $\frac{dy}{d\alpha} = \frac{\partial K}{\partial \alpha} \frac{\partial y}{\partial K} < 0$ , i.e. the difference between the price and the true probability decreases as  $\alpha$  increases.

Let  $S(\alpha, y)$  be the fraction of price-sensitive agents, as a function of the fraction of knowledgeable agents and the mispricing,  $S = \alpha + (\epsilon + y) \frac{1-\alpha}{\pi_r - \underline{\pi}}$ . Let  $y^*(\alpha)$  be the equilibrium mispricing, as a function of  $\alpha$ . Let  $S^*(\alpha) = S(\alpha, y^*(\alpha))$  be the fraction of price sensitive agents in equilibrium, as a function of alpha. Then,

$$\begin{aligned}S^*(\alpha) &= \alpha + \left( \epsilon + \sqrt{\left( \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) \right)^2 + \epsilon^2} - \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) \right) \frac{1-\alpha}{\pi_r - \underline{\pi}} = \\ &= \alpha + \frac{1-\alpha}{\pi_r - \underline{\pi}} \epsilon - \frac{1-\alpha}{\pi_r - \underline{\pi}} \frac{\alpha}{1-\alpha} (\pi_r - \underline{\pi}) + \frac{1-\alpha}{\pi_r - \underline{\pi}} \sqrt{\left( \frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi}) \right)^2 + \epsilon^2}\end{aligned}$$

$$\begin{aligned}
&= \frac{1-\alpha}{\pi_r - \underline{\pi}} \epsilon + \frac{1-\alpha}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} \\
&= \frac{1-\alpha}{\pi_r - \underline{\pi}} \left( \epsilon + \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} \right)
\end{aligned}$$

For any  $\alpha$  then we obtain  $S^*(\alpha)$  and  $y^*(\alpha)$ . If  $S^*(a)$  is strictly monotonic, we can invert it and obtain  $\alpha(S)$ . We are interested in  $y(\alpha(S))$  and  $\frac{dy}{dS} = \frac{dy}{d\alpha} \frac{d\alpha}{dS}$ . We know that  $\frac{dy}{d\alpha} < 0$ , hence we must only determine the sign of  $\frac{d\alpha}{dS}$ , which, under our conjecture that  $S^*(a)$  is strictly monotonic, coincides with the sign of  $\frac{dS^*(\alpha)}{d\alpha}$ . Figure 5 presents the surface of  $\frac{dS^*(\alpha)}{d\alpha}$  for any  $\alpha$  and any  $\epsilon$ , given  $\pi_r - \underline{\pi} = 0.5$ . It illustrates that if  $\epsilon$  is not too large relative to  $\alpha$ , the derivative is positive. We show that given our assumption that  $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ ,  $\frac{dS^*(\alpha)}{d\alpha} > 0$  for every  $\alpha$ , so that indeed  $S^*(a)$  is strictly monotonic as assumed, and it follows  $\frac{dy}{dS} < 0$ .

$$\frac{dS^*(\alpha)}{d\alpha} = -\frac{\epsilon}{\pi_r - \underline{\pi}} - \frac{1}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} + \frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2} \left( \left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2 \right)^{-1/2}.$$

We want to show that given any  $\epsilon < \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ ,

$$\begin{aligned}
-\frac{\epsilon}{\pi_r - \underline{\pi}} - \frac{1}{\pi_r - \underline{\pi}} \sqrt{\left(\frac{\alpha}{1-\alpha}(\pi_r - \underline{\pi})\right)^2 + \epsilon^2} + \frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)^2 \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2}} &> 0 \Leftrightarrow \\
\frac{\alpha(\pi_r - \underline{\pi})^2}{(1-\alpha)^2 \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2} - \epsilon &> 0 \Leftrightarrow \\
\frac{(\pi_r - \underline{\pi})}{(1-\alpha) \sqrt{1 + \left(\frac{(1-\alpha)\epsilon}{\alpha(\pi_r - \underline{\pi})}\right)^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1-\alpha)}\right)^2 + \epsilon^2} - \epsilon &> 0.
\end{aligned}$$

The left hand side expression is decreasing in  $\epsilon$ , hence it suffices to show that the inequality holds for  $\epsilon = \frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})$ .

$$\begin{aligned}
& \frac{(\pi_r - \underline{\pi})}{(1 - \alpha)\sqrt{1 + \left(\frac{(1-\alpha)\frac{\alpha}{1+\alpha}(\pi_r - \underline{\pi})}{\alpha(\pi_r - \underline{\pi})}\right)^2}} - \sqrt{\left(\frac{\alpha(\pi_r - \underline{\pi})}{(1 - \alpha)}\right)^2 + \frac{\alpha^2}{(1 + \alpha)^2}(\pi_r - \underline{\pi})^2} - \frac{\alpha}{1 + \alpha}(\pi_r - \underline{\pi}) > 0 \Leftrightarrow \\
& \frac{1}{(1 - \alpha)\sqrt{1 + \frac{(1-\alpha)^2}{(1+\alpha)^2}}} - \alpha\sqrt{\frac{1}{(1 - \alpha)^2} + \frac{1}{(1 + \alpha)^2}} - \frac{\alpha}{1 + \alpha} > 0 \Leftrightarrow \\
& \frac{(1 + \alpha)}{(1 - \alpha)\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2}} - \frac{\alpha}{(1 - \alpha)(1 + \alpha)}\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} - \frac{\alpha}{1 + \alpha} > 0 \Leftrightarrow \\
& (1 + \alpha)^2 - \alpha((1 + \alpha)^2 + (1 - \alpha)^2) - \alpha(1 - \alpha)\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} > 0 \Leftrightarrow \\
& (1 - \alpha)(1 + \alpha)^2 - \alpha(1 - \alpha)^2 > \alpha(1 - \alpha)\sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} \Leftrightarrow \\
& \frac{(1 + \alpha)^2}{\alpha} - (1 - \alpha) > \sqrt{(1 + \alpha)^2 + (1 - \alpha)^2} \Leftrightarrow \\
& \frac{(1 + \alpha)^4}{\alpha^2} + (1 - \alpha)^2 - 2\frac{(1 + \alpha)^2}{\alpha}(1 - \alpha) > (1 + \alpha)^2 + (1 - \alpha)^2 \Leftrightarrow \\
& (1 + \alpha)^2 - 2\alpha(1 - \alpha) > \alpha^2 \Leftrightarrow \\
& 1 + 2\alpha^2 > 0.
\end{aligned}$$

## B. Biased Slope Coefficients

To determine whether there is any simultaneous-equation bias on the estimated slope coefficients induced by overall balance in the changes in positions, we translate our setting into a more familiar framework, namely, that of a simple demand-supply setting. In particular, we are going to interpret (minus) the changes in endowments of the price-insensitive subjects as the supply in a demand-supply system with exogenous, price-insensitive supply, while the changes in endowments of the price-sensitive subjects correspond to the (price-sensitive) demands in a demand-supply system. The requirement that changes in holdings balance then corresponds to the usual restriction that demand equals supply.

We will consider only the case where price-sensitive subjects reduce their holdings when prices increase; translated into the usual demand-supply setting, this means that we assume that the slope of the demand equation is negative.

Assume there are only two subjects. One is price-sensitive, the other is price-insensitive. The former's changes in holdings corresponds to the demand  $\tilde{D}$  in the traditional demand-supply system; the latter's changes corresponds to the (exogenous) supply  $\tilde{S}$ . The usual assumptions are as follows:

$$\tilde{D} = A + BP + \epsilon,$$

with  $B < 0$ , and

$$\tilde{S} = \eta,$$

where  $\epsilon$  is mean zero, and is independent of  $\eta$ .  $P$  denotes price.

We want to know the properties of the OLS estimate of  $B$ . Assume that  $P$  is determined by equating demand and supply (equivalent to balance between changes in holdings), i.e., from

$$\tilde{D} = \tilde{S}.$$

Then:

$$\text{cov}(P, \epsilon) = -\frac{1}{B} \text{var}(\epsilon) > 0.$$

Because of this, standard arguments show that the OLS estimate of  $B$  is inconsistent, with an upward bias. As such, the nominal size of the usual  $t$ -test under-estimates the true size, and one should apply a generous cut-off in order to determine whether  $B$  is significantly negative.

In our case, however, we only need to identify who is price-sensitive (i.e., whose holdings changes correspond to  $D$  in the demand-supply setting?) and who is not (whose holdings changes correspond to  $\tilde{S}$ ?). For this, we just run an OLS projection of changes in endowments on prices. The subjects with significantly negative slope coefficients are price-sensitive and hence, map into the demand  $\tilde{D}$  of the traditional demand-supply system. The argument above, however, indicated that this test is biased. Therefore, a generous cut-off should be chosen; we chose a cut-off equal to 1.6.

While we did not need this for our study, one can obtain an improved estimate of the price sensitivity once subjects are categorized as either price-sensitive or price-insensitive. Indeed, the changes in the holdings of the price-insensitive subjects can be used as instrument to re-estimate the price-sensitivity of the price-sensitive subjects. This is equivalent to using  $\tilde{S}$  as an instrument to estimate  $B$ . Indeed,  $\tilde{S}$  ( $= \eta$ ) and  $\epsilon$  are uncorrelated, while  $\tilde{S}$  and  $P$  are correlated ( $\text{cov}(\tilde{S}, P) = \text{var}(\tilde{S})/B$ ), so  $\tilde{S}$  is a valid instrument to estimate  $B$  in standard instrumental-variables analysis.

# Instructions

## I. THE EXPERIMENT

**1. Situation** The experiment consists of a sequence of trading sessions, referred to as *periods*. At the beginning of *even-numbered* periods, you will be given a fresh supply of *securities* and *cash*; in *odd-numbered periods*, you carry over securities and cash from the previous period. Markets open and you are free to trade some of your securities. You buy securities with cash and you get cash if you sell securities.

At the end of *odd-numbered* periods, the securities expire, after paying *dividends* that will be specified below. These dividends, together with your cash balance, constitute your *period earnings*. Securities do not pay dividends at the end of even-numbered periods and cash is carried over to the subsequent period, so your period earnings in even-numbered periods will be zero.

Period earnings are *cumulative* across periods. At the end of the experiment, the cumulative earnings are yours to keep, in addition to a standard sign-up reward.

During the experiment, accounting is done in real dollars.

**2. The Securities** You will be given two types of securities, *stocks* and *bonds*. Bonds pay a fixed dividend at the end of a period, namely, \$0.50. Stocks pay a random dividend. There are two types of stocks, referred to as Red and Black. Their payoff depends on the drawing from a deck of 4 cards, as explained later. The payoff is either \$0.50 or nothing. When Red stock pays \$0.50, Black stock pays nothing; when Red stock pays nothing, Black stock pays \$0.50.

You will be able to trade Red stock as well as bonds, but not Black stock.

You won't be able to buy Red stock or bonds unless you have the cash. You will be able to sell Red stock and bonds (and get cash) even if you do not own any. This is called short selling. If you sell, say, one Red stock, then you get to keep the sales price, but \$0.50 will be subtracted from your period earnings after the market closes and if the payoff on Red stock is \$0.50. If at the end of a period you are holding, say, -1 bonds, \$0.50 will be subtracted from your period earnings.

The trading system checks your orders against bankruptcy: you will not be able to submit orders which, if executed, are likely to generate negative period earnings.

**3. How Payoffs Are Determined** Each period, we start with a deck of 4 cards: one hearts ( $\heartsuit$ ), one diamonds ( $\diamondsuit$ ), one clubs ( $\clubsuit$ ) and one spades ( $\spadesuit$ ). The cards are shuffled and put in a row, face down.

Our computer takes randomly one or two cards and it discards them.

From the remaining cards, our computer randomly picks one or two cards. If one of these cards is hearts ( $\heartsuit$ ), then the computer puts it back and picks another one. Sometimes, the computer will even put back diamonds ( $\diamondsuit$ ) and pick another one. The computer then reveals the card(s) it picked and we will announce this in the News Page at the end of the period (after that, another period starts with the same securities in which you can trade again). *Note that the revealed card(s) will never be hearts, and sometimes may not even be diamonds.*

Before each period, the News Page will provide all the information that you need to make the right inferences: (i) whether one or two cards are going to be discarded initially, (ii) whether one or two cards are going to be picked from the remaining cards and whether diamonds will ever be shown.

After we show the revealed cards, one or two cards remain in the deck. Our computer randomly picks a card and *this last card determines the payoff on the securities.*

Red stock pays \$0.50 when the last card is either hearts ( $\heartsuit$ ) or diamonds ( $\diamondsuit$ ). In those cases, the Black stock pays nothing. This is shown in the following Payoff Table.

	Red Stock	Black Stock
If last card is $\heartsuit$ or $\diamondsuit$	\$0.50	0
If last card is $\clubsuit$ or $\spadesuit$	0	\$0.50

Here is an example. Initially there are 4 cards in the deck, randomly shuffled. They are put in a row, face down, like this:

□ □ □ □

Our computer randomly discards one card (the third one in this case):

□ □ □

Our computer then randomly picks one card (the fourth one in this case), and reveals it, provided it is not hearts or diamonds (in this case; if it is hearts or diamonds, it replaces it with another card from the deck that is neither):

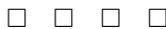


From the remaining two cards, our computer picks one at random that determines the payoffs on the stocks.



In this case, the last card picked is diamonds. As a result, you would be paid \$0.50 for each unit of Red stock you're holding, and nothing for the Black stock you're holding.

Here is another example. Initially there are 4 cards in the deck, randomly shuffled. They are put in a row, face down:



Our computer randomly discards two cards (the second and third ones in this case):



Our computer then randomly picks one card and reveals it, provided it is not hearts (if it is hearts, it replaces it with another card from the deck):



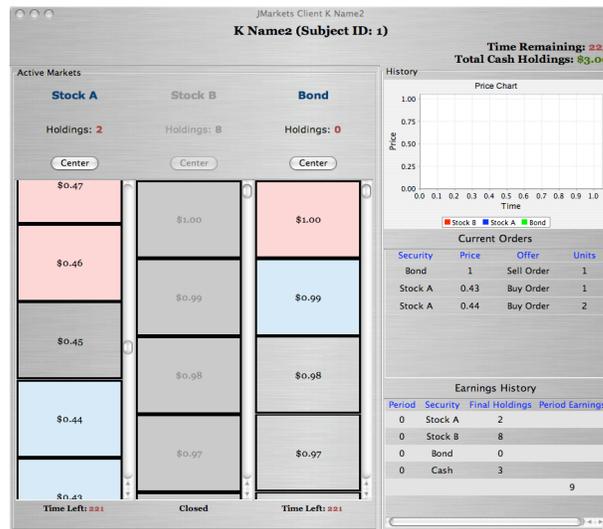
Our computer then picks the remaining card, which determines the payoffs on the stocks.



In this case, the last card picked is hearts. As a result, you would be paid \$0.50 for each unit of Red stock you're holding, and nothing for the Black stock you're holding.

Again, the announcements of the number of cards that will be discarded initially and revealed at the end of the period can be found in the News page. This page will also display the card(s) that are turned over at the end of the period, and, at the end of the subsequent period, the final card that determines the payoff on the Stocks.

**II. THE MARKETS INTERFACE, jMARKETS** Once you click on the Participate link to the left, you will be asked to log into the markets, and you will be connected to the jMarkets server. After everybody has logged in and the experiment is launched, a markets interface like the one below will appear



### 1. Active Markets

The Active Markets panel is renewed each period. In it, you'll see several scroll-down columns. Each column corresponds to a market in one of the securities. The security name is indicated on top. At the bottom, you can see whether the market is open, and if so, how long it will remain open. The time left in a period is indicated on the right hand side above the Active Markets panel.

At the top of a column, you can also find your current holdings of the corresponding security. Your current cash holdings are given on the right hand side above the Active Markets panel.

Each column consists of a number of price levels at which you and others enter offers to trade. Current offers to sell are indicated in red; offers to buy are indicated in blue. When pressing the Center button on top of a column, you will be positioned halfway between the best offer to buy (i.e., the highest price at which somebody offers to buy) and the best offer to sell (i.e., the lowest price that anybody offers to sell at).

When you move your cursor to a particular price level box, you get specifics about the available offers. On top, at the left hand side, you'll see the number of units requested for purchase. Each time you click on it, you send an order to buy one unit yourself. On top, at the right hand side, the number

of units offered for sale is given. You send an order to sell one unit each time you yourself click on it. At the bottom, you'll see how many units you offered. (Your offers are also listed under Current Orders to the right of the Active Markets panel.) Each time you hit cancel, you reduce your offer by one unit.

If you click on the price level, a small window appears that allows you to offer multiple units to buy or to sell, or to cancel offers for multiple units at once.

## **2. History**

The History panel shows a chart of past transaction prices for each of the securities. Like the Active Markets panel, it refreshes every period. jMarkets randomly assigns colors to each of the securities. E.g., it may be that the price of the Red Stock is shown in blue. Make sure that this does not confuse you.

## **3. Current Orders**

The Current Orders panel lists your offers. If you click on one of them, the corresponding price level box in the Active Markets panel is highlighted so that you can easily modify the offer.

## **4. Earnings History**

The Earnings History table shows, for each period, your final holdings for each of the securities (and cash), as well as the resulting period earnings.

## **5. How Trade Takes Place**

Whenever you enter an offer to sell at a price below or equal to that of the best available buy order, a sale takes place. You receive the price of the buy order in cash. Whenever you enter an offer to buy at a price above or equal to that of the best available sell order, a purchase takes place. You will be charged the price of the sell order.

The system imposes strict price-time priority: buy orders at high prices will be executed first; if there are several buy orders at the same price level, the oldest orders will be executed first. Analogously, sell orders at low prices will be executed first, and if there are several sell orders at a given price level, the oldest ones will be executed first.

## **6. Restrictions On Offers**

Before you send in an offer, jMarkets will check two things: the cash constraint, and the bankruptcy constraint.

The cash constraint concerns whether you have enough cash to buy securities. If you send in an offer to buy, you need to have enough cash. To allow you to trade fast, jMarkets has an automatic cancelation feature. When you submit a buy order that violates the cash constraint, the system will automatically attempt to cancel buy orders you may have at lower prices, until the cash constraint is satisfied and your new order can be placed.

The bankruptcy constraint concerns your ability to deliver on promises that you implicitly make by trading securities. We may not allow you to trade to holdings that generate losses in some state(s). A message appears if that is the case and your order will not go through.

## Tables and Figures

**Table I**  
**Parameters in the Experimental Design**

Experiment <sup>a</sup>	Subject	Signup	Initial Allocations <sup>b</sup>			
	Category (Number)	Reward (Dollar)	Red Stock (Units)	Black Stock (Units)	Notes (Units)	Cash (Dollar)
Panel A						
Sessions with no aggregate risk						
Caltech	10	5	0	9	0	4
	10	5	12	3	0	1
Utah-1	10	5	0	9	0	4
	10	5	12	3	0	1
Caltech-Utah-1	10	5	0	9	0	4
	10	5	12	3	0	1
UCLA	10	5	0	9	0	4
	10	5	12	3	0	1
Utah-2	10	5	0	9	0	4
	10	5	12	3	0	1
Caltech-Utah-2	10	5	0	9	0	4
	10	5	12	3	0	1
Panel B						
Sessions with aggregate risk						
UCLA-R1	11	5	0	9	0	4
	9	5	12	3	0	1
UCLA-R2	11	5	0	9	0	4
	9	5	12	3	0	1
UCLA-R3	11	5	0	9	0	4
	9	5	12	3	0	1

<sup>a</sup>Indicates affiliation of subjects. “Utah” refers to the University of Utah; “Utah-Caltech” refers to: 50% of subjects were Caltech-affiliated; the remainder were students from the University of Utah. Experiments are listed in chronological order of occurrence.

<sup>b</sup>Renewed each period.

**Table II**  
**Treatments**

Treatment	Periods	Number of Cards Discarded Initially	Number of Cards Revealed Half-time	Cards Never Revealed Half-time
1	1, 5	1	2	hearts
2	2, 7	1	1	hearts
3	3, 6	2	1	hearts
4	4, 8	1	1	hearts, diamonds

**Table III**  
**Price Sensitivity**

Experiment	Treatment	I	II	III	IV
		Mean Absolute Mispricing, $M^a$	# of $(T_b < -1.65)$ Subjects, $N_{(T_b < -1.65)}^b$	# of $(T_b > 1.9)$ Subjects, $N_{(T_b > 1.9)}$	# of $(T_b < -1.65)$ $T_a$ -correctly signed $N_{(T_b < -1.65, T_a)}^c$
Panel A					
Sessions with no aggregate risk					
Caltech	1	3.13	7	2	6
	2	5.54	3	2	2
	3	3.4	2	2	1
	4	1.25	4	6	2
Utah-1	1	3.5	4	1	3
	2	11.79	1	2	0
	3	11.28	0	1	0
	4	7.24	1	0	0
Caltech-Utah-1	1	3.35	5	1	1
	2	4.75	2	0	0
	3	1.62	2	0	2
	4	1.89	5	3	3
UCLA	1	4.86	2	1	1
	2	5.07	3	0	1
	3	2.53	3	3	3
	4	2.9	3	0	2
Utah-2	1	7.5	2	0	1
	2	7.95	3	3	0
	3	5.88	6	2	1
	4	4.19	2	2	2
Caltech-Utah-2	1	8.78	1	0	0
	2	2.58	2	0	2
	3	2.77	3	1	1
	4	3.38	3	0	0
Panel B					
Sessions with aggregate risk					
UCLA-1R	1	2.02	0	1	0
	2	10.91	0	1	0
	3	4.87	4	1	4
	4	3.64	0	1	0
UCLA-2R	1	3.91	3	2	1
	2	12.30	1	0	0
	3	8.79	5	4	2
	4	6.60	2	2	1
UCLA-3R	1	6.43	2	0	1
	2	6.06	3	2	1
	3	2.66	3	2	2
	4	3.30	1	0	0

$$\text{Corr}(M, N_{(T_b < -1.65)}) = -0.53^d$$

(St. Error = 0.146)

$$\text{Corr}(M, N_{(T_b < -1.65, T_a)}) = -0.58^e$$

(St. Error = 0.088)

<sup>a</sup>In U.S. cents.

<sup>b</sup> $T_b$  is the t-statistic of the slope coefficient estimate in projections of one-minute changes in individual holdings of Red Stock onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

<sup>c</sup> $T_a$  is the t-statistic of the intercept in projections of one-minute changes in individual holdings of Red Stock onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities.

<sup>d</sup>For completeness, when the sessions with aggregate risk are included,  $\text{Corr}(M, N_{(T_b < -1.65)}) = -0.37$

<sup>e</sup>When the sessions with aggregate risk are included,  $\text{Corr}(M, N_{(T_b < -1.65, T_a)}) = -0.45$

**Table IV**  
**Price Sensitivity and Imbalance Relation**

The table presents the slope coefficients from the (two-level) regression of individual imbalances  $I_i$ , where  $i$  indexes the subjects, on parameters  $X_i$  that incorporate the measurement of price-sensitivity of subject  $i$ :

$$I_i = a + b_{between}X_i + \epsilon_i, \text{ where}$$

$$I_{ij} = I_i + \eta_{ij},$$

$$X_{ij} = X_i + \xi_{ij},$$

$$\eta_{ij} = b_{within}\xi_{ij} + \delta_{ij},$$

where  $j$  indexes the treatments  $j \in \{1, 2, 3, 4\}$  in which subject  $i$  participated. The first row of the table reports from regressions with  $X_{ij} = T_{ij}$  ( $T_{ij}$  is the t-statistic of the slope coefficient estimate in projections of one-minute changes in individual  $i$ 's holdings of Red Stock in Treatment  $j \in \{1, 2, 3, 4\}$  onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities). The second row corresponds to  $X_{ij} = MT_{ij} = M_{ij}T_{ij}$ , where  $M_{ij}$  is the absolute mispricing in treatment  $j$  of the session in which subject  $i$  participated. Standard errors (reported in parentheses) in all projections are corrected for heteroscedasticity and subject clustering.

Independent Variable		Panel A		Panel B		Panel C	
		No Aggregate Risk		Aggregate Risk		Both Risk Treatments	
		End-period	Mid-Period	End-period	Mid-Period	End-Period	Mid-Period
$T$	Coef.	-11.287	-10.518	-16.477	-14.557	-12.985	-12.083
	St. Error	(22.399)	(17.244)	(4.396)	(3.579)	(6.299)	(5.054)
	$R^2$	0.050	0.068	0.342	0.422	0.148	0.197
$MT$	Coef.	-0.730	-1.986	-2.898	-2.499	-1.818	-2.230
	St. Error	(1.323)	(2.026)	(0.763)	(0.626)	(0.764)	(0.777)
	$R^2$	0.015	0.114	0.359	0.489	0.119	0.265

**Table V**  
**Price Sensitivity and Number of Trades Relation**

The table presents the slope coefficients from the (two-level) regression of individual number of trades  $NT_i$ , where  $i$  indexes the subjects, on parameters  $X_i$  that incorporate the measurement of price-sensitivity of subject  $i$ :

$$NT_i = a + b_{between}X_i + \epsilon_i, \text{ where}$$

$$NT_{ij} = NT_i + \eta_{ij},$$

$$X_{ij} = X_i + \xi_{ij},$$

$$\eta_{ij} = b_{within}\xi_{ij} + \delta_{ij},$$

where  $j$  indexes the treatments  $j \in \{1, 2, 3, 4\}$  in which subject  $i$  participated. The first row of the table reports from regressions with  $X_{ij} = T_{ij}$  ( $T_{ij}$  is the t-statistic of the slope coefficient estimate in projections of one-minute changes in individual  $i$ 's holdings of Red Stock in Treatment  $j \in \{1, 2, 3, 4\}$  onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities). The second row corresponds to  $X_{ij} = MT_{ij} = M_{ij}T_{ij}$ , where  $M_{ij}$  is the absolute mispricing in treatment  $j$  of the session in which subject  $i$  participated. Standard errors (reported in parentheses) in all projections are corrected for heteroscedasticity and subject clustering.

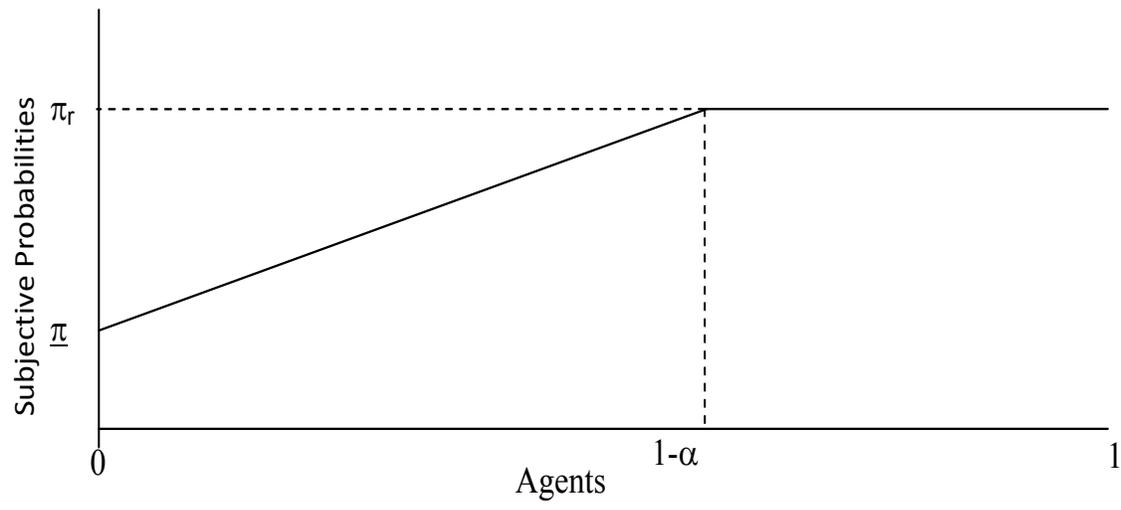
Independent Variable		Panel A		Panel B		Panel C	
		No Aggregate Risk		Aggregate Risk		Both Risk Treatments	
		End-period	Mid-Period	End-period	Mid-Period	End-Period	Mid-Period
$T$	Coef.	0.307	-6.184	-10.218	-7.481	-7.624	-6.976
	St. Error	(22.349 )	(22.654 )	(6.846 )	(4.446 )	(8.866 )	(6.058 )
	$R^2$	0.000	0.012	0.095	0.096	0.021	0.043
$MT$	Coef.	1.355	-0.217	-2.064	-1.244	-0.693	-0.936
	St. Error	(2.682 )	(2.538 )	(1.523 )	(1.026 )	(1.398 )	(1.024 )
	$R^2$	0.015	0.001	0.131	0.103	0.007	0.029

**Table VI**  
**Price Sensitivity and Imbalance/Number of Trades Relation, OLS Regression Results**

The first (second) part of the table presents the slope coefficients from the projections of individual imbalances  $I_{ij}$  (individual number of trades  $NT_{ij}$ ), where  $i$  indexes subjects, while  $j$  indexes treatments,  $j \in \{1, 2, 3, 4\}$ , onto parameters  $X_{ij}$  that incorporate the measurement of price-sensitivity of subject  $i$  in treatment  $j$ :  $I_{ij} = a + bX_{ij} + \epsilon_{ij}$  ( $NT_{ij} = a + bX_{ij} + \epsilon_{ij}$ ). The first row of each section reports from regressions with independent variable of  $X_{ij} = T_{ij}$  ( $T_{ij}$  is the t-statistic of the slope coefficient estimate in projections of one-minute changes in individual  $i$ 's holdings of Red Stock in Treatment  $j \in \{1, 2, 3, 4\}$  onto difference between (i) last traded price, and (ii) expected payoff using correct probabilities). The second row corresponds to  $X_{ij} = MT_{ij} = M_{ij}T_{ij}$ , where  $M_{ij}$  is the absolute mispricing in treatment  $j$  of the session in which subject  $i$  participated. Standard errors (reported in parentheses) in all projections are corrected for heteroscedasticity and subject clustering.

Price Sensitivity and Imbalance Regression, OLS Results						
Independent Variable	Panel A		Panel B		Panel C	
	No Aggregate Risk		Aggregate Risk		Both Risk Treatments	
	End-period	Mid-Period	End-period	Mid-Period	End-Period	Mid-Period
$T$	-0.064 (0.785)	-0.406 (0.662)	-2.728 (0.755)	-2.054 (0.698)	-0.716 (0.614)	-0.814 (0.525)
$MT$	0.031 (0.088)	-0.059 (0.078)	-0.395 (0.107)	-0.257 (0.087)	-0.115 (0.073)	-0.133 (0.061)

Price Sensitivity and Number of Trades Regression, OLS Results						
Independent Variable	Panel D		Panel E		Panel F	
	No Aggregate Risk		Aggregate Risk		Both Risk Treatments	
	End-period	Mid-Period	End-period	Mid-Period	End-Period	Mid-Period
$T$	-0.384 (1.107)	-0.334 (0.889)	-2.342 (1.205)	1.238 (0.821)	-0.967 (0.873)	-0.601 (0.693)
$MT$	-0.004 (0.181)	0.023 (0.104)	-0.326 (0.215)	-0.115 (0.123)	-0.125 (0.140)	-0.032 (0.079)



**Figure 1.** Beliefs

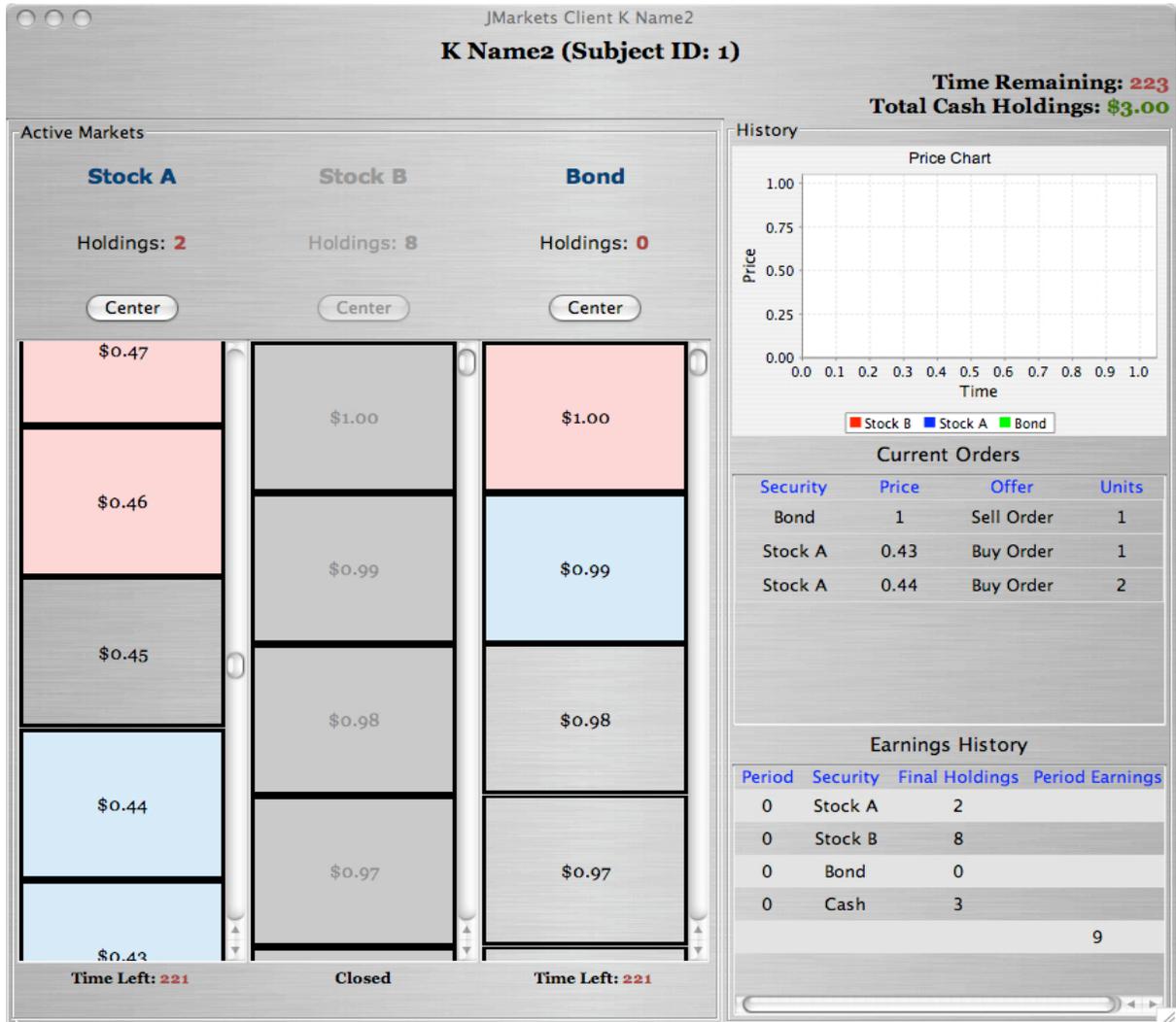
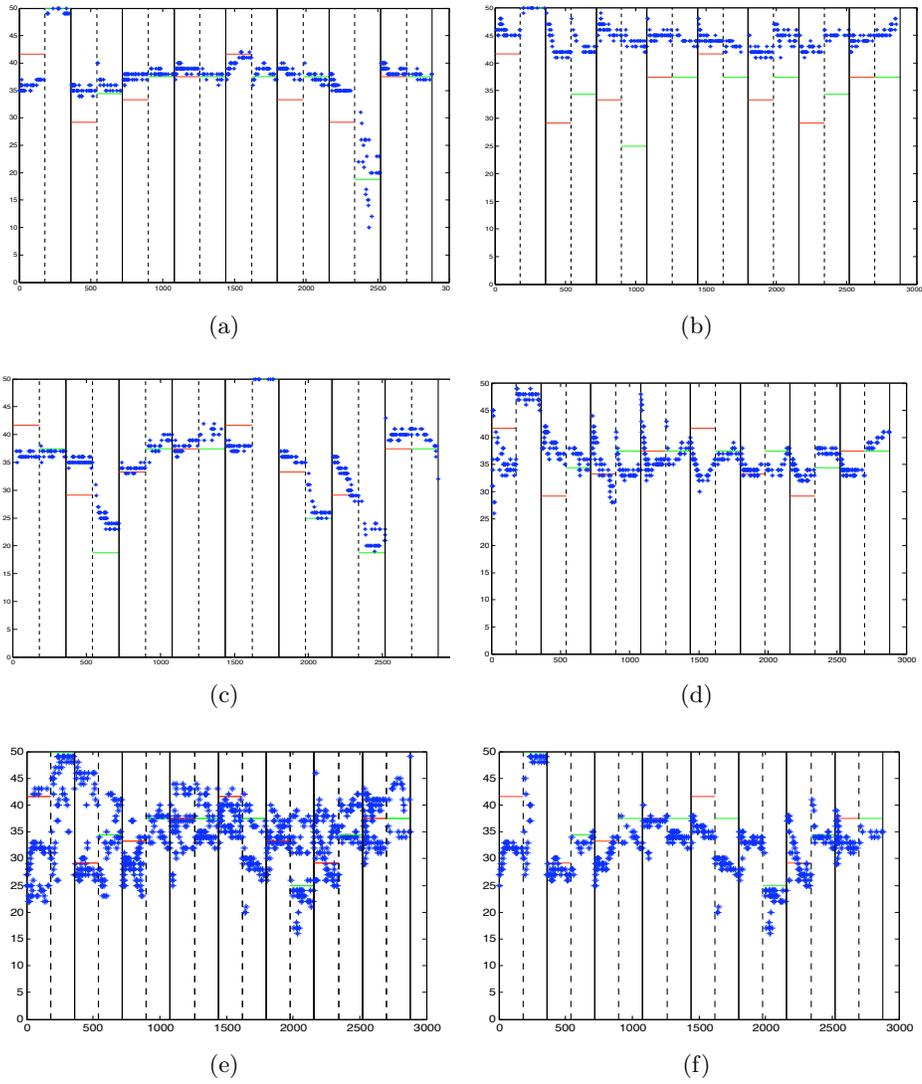
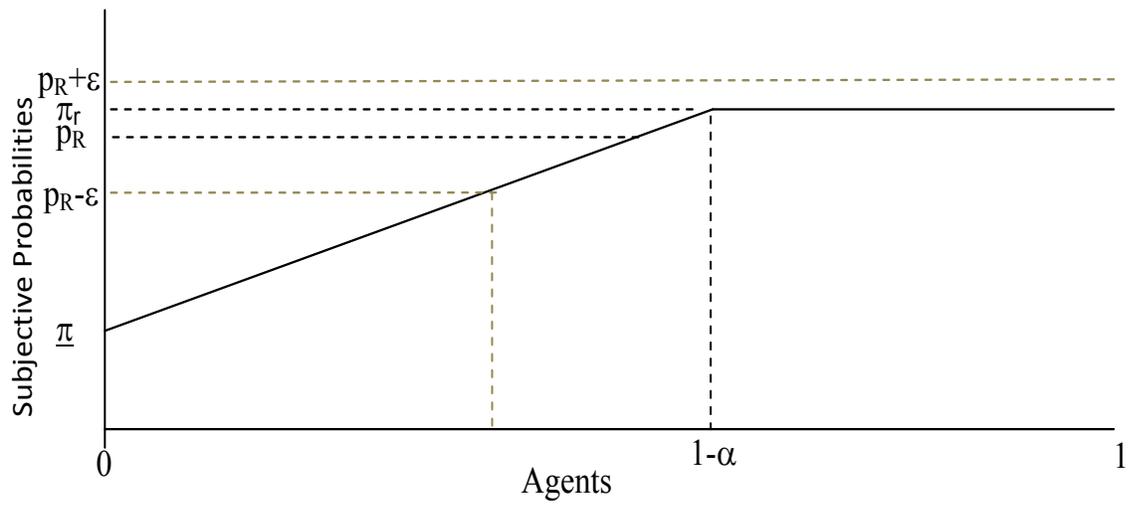


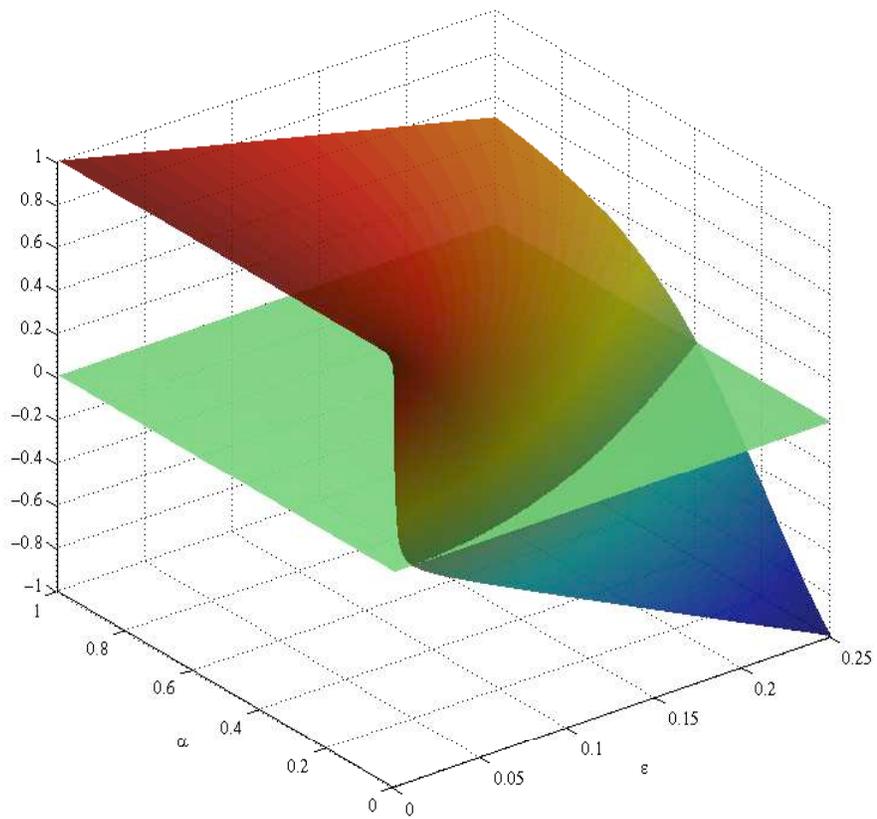
Figure 2. jMarkets Trading Screen



**Figure 3.** Transaction Prices: (a) Caltech; (b) Utah-1; (c) Caltech-Utah-1; (d) UCLA; (e) Utah-2; and, (f) Caltech-Utah-2.



**Figure 4.** Equilibrium



**Figure 5.**  $\frac{\partial S}{\partial \alpha}$  as a function of  $\alpha$  and  $\epsilon$  when  $\pi_r - \underline{\pi} = 0.5$ .