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FIRM UNDER BOTH OUTPUT AND LABOUR-MARKET
UNCERTAINTY

ALLOCATION AND DISTRIBUTION IN AN EMPLOYEE-OWNED FIRM UNDER BOTH OUTPUT AND LABOR MARKET UNCERTAINTY

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and
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ABSTRACT

This paper examines the effects of risk in output and labor markets on membership, employment, layoff and layoff compensation determination in a risk-averse employee-owned firm. Expected incomes of working and furloughed member owners are shown to be equal only when income variances are equal; otherwise the group subject to the higher income variance receives higher mean income. The extent to which average member incomes exceed value marginal product is shown to be related linearly to price variance and risk aversion. Comparative static implications of effects of changes in price and unemployment income means and variances among others, are derived. The results are related to previous theoretical and empirical findings on the labor-managed firm, and found to strongly qualify previous studies.

1. INTRODUCTION

The level of uncertainty in labor markets in Europe and elsewhere has been increasing markedly since the mid-1970s. The increasing openness of OECD economies has in many cases led to the decline of national industries previously regarded as offering stable employment. The end of central planning in East Europe has brought higher unemployment and risk of future joblessness along with prospects of efficiency and an end to technological stagnation. And unemployment has risen at the same time that the welfare state in many countries

has been unable to provide as strong a "safety net" as society had come to expect. Thus, the variances of both wages and unemployment benefits has risen.

In the same period, employment in employee-owned firms (EOFs) has increased dramatically. In a major survey in this journal, Estrin (1985) summarizes evidence from European Commission studies that in Europe both the number of 100% EOFs and the number of workers they employ roughly doubled from the late 1970s to the mid-1980s, to about half a million workers in about 14,000 EOFs (see also European Communities, 1986). Throughout the late 1980s, this sector appears to be continuing to experience growth though at a somewhat less rapid pace. Many Italian cooperatives have waiting lists as large as their working memberships.[1] Conversions of conventional firms to worker-owned firms have become common in the United States (Bradley and Gelb, 1983). The number of employee-owned and cooperative firms in East Europe has shown a huge upsurge in 1989—90 in response both to the new market freedoms and actual or anticipated layoffs. LMFs have also had to furlough workers in the 1980s; yet they have also affered some "safety net" protections as they have provided compensation to laid-off members.

Among other things, these developments raise issues for economic analysis. Workers in these firms, whether traditional European producer cooperatives or new forms with direct employee holdings of varying numbers of transferable shares, hold some form of property rights to their job or compensation for being furloughed. Thus the Wardian flexible employment model is not applicable. Among the questions of interest are: How is the level of compensation for laid-off members related to the mean and variance of inside and outside income? How is the behavior of the employee-owned sector affected by increased unemployment (lower expected incomes in alternative employment), or an increase in the variance of alternative income? How do such changes affect the level of active employment, expected income for working members, and contractual compensation for furloughed members? What is the effect of a change in the variance of the firm's output price on employment and compensation? In what ways does the economic behavior of a more risk-averse EOF differ from that of a less risk-averse one? What role does the size of total membership play?

These are some of the questions which we are able to address with the formal analysis which we begin in the following section.

The paper is organized as follows. In Section 2, we develop the assumptions of the study and set up the basic model. We carefully examine the major assumptions which enable us to carry out our analysis, and demonstrate that they bear a closer resemblance to observed producer cooperative and EOF institutions — and build more systematically on recent developments in the economics of uncertainty

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[1] Interviews with officials of the Lega Nazionale delle Cooperative e Mutue, Roma and Bologna, June 1984 and April—May 1990.

— than do previous papers in this field. We then solve the expected utility maximization program of a fully EOF facing both output and labor market uncertainty.

In Section 3, we demonstrate that the expected income of working members is greater than or less than the sum of expected alternative monetary equivalent income (EAMEI) plus paid compensation to furloughed members as the variance of working member's incomes is greater than or less than the variance of the EAMEI of furloughed members. Expected incomes of working and furloughed members are equal only when income variances are equal. We show that this finding offers important qualifications to most of the previous work on the EOF under uncertainty.

We derive a condition demonstrating the roles of technology, price variance and risk aversion in the optimal solution. This enables us to predict the extent to which average incomes will exceed the marginal productivity of labor; this inequality has been observed in the empirical literature but is not predicted by the standard LMF model.

In Section 4, we derive conditions determining the employment, expected income and unemployment compensation response to changes in expected price and its variance, EAMEI and its variance, fixed costs, and the degree of risk aversion. Some of these comparative static questions are new to the paper and others demonstrate that results in the earlier literature are strongly dependent on restrictive assumptions.

In Section 5, we offer concluding remarks and suggest a number of extensions and interpretations.

2. ASSUMPTIONS AND SET-UP OF THE MODEL

In this section, we set up the EOF's problem of contracting over employment rules and compensation in the event of layoffs. Assumptions are developed and interpreted. The economic necessity for the EOF to offer an ex ante unemployment compensation contract rather than a state contingent contract is stressed. Since the alternative income of laid - off workers is uncertain, it is argued that such a non-state-contingent contract is the most appropriate formulation for the study of these problems. The relevant Kuhn—Tucker conditions characterizing the allocational decisions of the EOF are derived.

Each employed member receives the same income. We define the income per currently employed member, y, as

$$y = \frac{pf - F - (L - l) \cdot c}{l} \tag{1}$$

where

    p is the output price of the LMF;
    L is the membership level;
    l is the number of working members;

f(l) is the production function, where it is assumed that
f(l) = 0 for $0 < l \leq l$ and f(l) > 0, f'(l) > 0, and f"(l) < 0, for $l > l$;
F is the fixed non-labor cost; and
c is the contractual compensation for furloughed members.

Six assumption are crucial to our analysis; each will be stated formally and considered in its relationship to previous literature. In each case, its present economic meaning is examined.

A.1. Total membership, L, is taken as fixed for the present period of analysis. It is widely argued in the EOF literature that membership is no more flexible than capital (Vanek (1970)).

A.2. Employment, l, is neither instantaneously nor costlessly adjustable. In particular, let the EOF contract to pay compensation, c to furloughed members.

A.2 postulates our view that the "Illyrian" literature on the behavior of the EOFunder uncertainty[2] is inapplicable to actual EOFs. In this literature, each time a price fluctuation is observed, employment is adjusted instantaneously and costlessly to maximize instantaneous post-layoff working members' utility. The Illyrian approach fails to address the obvious fact that members in an EOF hold property rights and/or take part in democratic voting over policy; these rights include appropriate compensation when some of the property rights are lost. Some of the recent literature on the LMF under both certainty and uncertainty has begun to formalize such rights, or to examine expected income or utility maximization over working and non-working members, or both;[3] we wish to build on this work in modeling actual PC and other EOF institutional characteristics.

A.1 also implies an ex ante determination of layoff compensation, and not a state contingent contract.[4] Our reformulation more realistically captures the economics of the problem; and in fact is more in tune with observed institutions. It also has the advantage of leading to testable theorems on compensation contracts as well as to new comparative static results. The EOF cannot observe and measure its members' outside utility and so cannot contract for unemployment benefits which ensure a level of unemployment utility given the state. It can at best estimate the mean and variance of outside utility, and base a contractually-fixed unemployment compensation on these parameters. (Compensation can change when these *parameters* are observed to change, but not each time a random variable takes a particular

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[2] The papers in this literature include Ramachandran, Russell, and Seo (1979), Muzondo (1979), Pestieau (1979), Bonin et al (1980), Paroush and Kahana (1980), Hey (1981), and Hawawini and Michel (1981). In this case an "Illyrian" firm refers to one maximizing the expected utility of income per worker without compensation or property rights for laid-off members (for a complete survey comparing this type of firm with profit maximizing and joint stock firms, see Hey (1981).

[3] See Steinherr and Thisse (1979), Bonin (1981), Svejnar (1982), Brewer and Browning (1982), Miyazaki and Neary (1983), Miyazaki (1984), McCain (1985) and Smith and Ye (1987).

[4] Examples of state-contingent contracts are found in Miyazaki (1984) and Miyazaki and Neary (1985).

value.) Even outside income of furloughed members is prohibitively expensive to monitor continously. To provide an illustration from the labor market as a whole, in most OECD countries unemployment compensation is based on a fraction of the wage received *at the time of layoff*, not the wage being received by working colleagues at the time each check is issued. Some income from part-time employment is often allowed with only partial or no penalty. In other words, compensation is determined by an ex ante contract rather than a state contingent one. In sum, though EOF may in principle offer state-contingent furlough pay, they can never offer state contingent EAMEI. And utility, not income, is to be maximized.

A.3 All members are identical and have a utility function, u, defined over total income (including EAMEI), $\dot{\omega}$. It is assumed that $u(\dot{\omega})$ exhibits constant Arrow—Pratt risk aversion, r:

$$r = -u''(\dot{\omega})/u'(\dot{\omega}).$$

The CARA assumption is standard for this type of analysis. It may be dropped only at the cost of assumption A.1, which we have argued is fundamental to the idea and practice of an EOF.

A.4 The output price p, is a random variable with $E(p) = \mu_p$ and $var(p) = \sigma_{p2}$, following a truncated normal distribution:

$$\phi(p) = exp\left(-(p-\mu_p)^2 / 2\sigma^2_p\right) / \sqrt{2\pi}\,\sigma_p, \text{ for } p \in (0, \bar{p}),$$
$$= 0, \text{ for } p \notin (0, \bar{p}),$$

where $\bar{p}$ is the upper bound of p, and σ_p is small enough such that the integral of $\phi(p)$ over zero to $\bar{p}$ is approximately equal to 1.

The important implication of A.4 is that the price distribution is completely characterized by the first two moments, a standard assumption.

Let v stand for the expected alternative monetary-equivalent income (EAMEI) for a laid-off member. In order to address the basic questions which we raised at the outset of the paper, we must allow for the important fact that v is a random variable, rather than known with certainty. To do so we adopt the following assumption.

A.5 v is a random variable with $E[v] = \mu_v$ and $var(v) = \sigma^2_v$ and follows a truncated normal distribution:

$$g(v) = exp\left(-(v-\mu_v)^2 / 2\sigma^2_v\right) / \sqrt{2\pi}\,\sigma_v, \text{ for } v \in (0, \bar{v}),$$
$$= 0 \text{ for } v \notin (0, \bar{v}),$$

where $\bar{v}$ is some upper bound of v, and σ_v is small enough such that the integral of g(v) over zero to $\bar{v}$ is approximately equal to 1.

The use of this distribution is standard in this type of analysis. A truncated normal distribution is widely regarded as more realistic than an unbounded normal distribution for most economic applica-

tions. Moreover, with regard to outside income at least, it is readily testable empirically.[5] Together with the CARA assumption, A.4 and A.5 enable us to carry out analysis.[6]

Finally, we have the following conventional assumption (see, for example, Brewer and Browning (1982)):

A.6 The LMF uses a purely random process to determine which members are to be laid off.

The last assumption, A.6, is consistent which A.3. All members are identical, so each should face the same probability of being laid off, which is $(L-l)/L$. The assumption is also formally consistent with a rotational layoff agreement. Now, the problem of the EOF is to choose the employment level, l, and the unemployment compensation, c, given the exogenous distribution of p and v and the predetermined membership, L, to maximize the expected utility of each member's monetary-equivalent income, or

$$\max_{c, l} (l/L) \, E_p \, [u(y)] + ((L-l)/L) \, E_v \, [u \, (v + c)] \tag{2}$$

$$s \cdot t \cdot l \leq L$$

After some algebra,[7] the first order conditions of the interior solution in which the employment constraint, $l < L$, is nonbinding, lead to:

$$E_p \, [u' \, (y)] = E_v \, [u' \, (v + c)] \tag{3}$$

and,

$$E_p \, [u' \, (y) \, (pf' + c - y)] = 0. \tag{4}$$

If $1 = L$, that is, the constraint is binding, it will be optimal to employ all members in the firm. Alternatively, this formulation provides us with the long-run determination of total membership size. The problem is reduced to maximizing expexted utility of income over total membership, or

$$\max_{L} E \, [u(y)] \, , \, where \, y = (pf(L) - F)/L \tag{5}$$

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[5] For example of econometric studies using panel data from Western employee-owned firms see Smith (1984), Jones and Svejnar (1985), Estrin, Jones and Svejnar (1987), Conte and Svejnar (1988).

[6] Solving $-u''(y)/u'(y) = r$ for u with a constant r, we have $u - ke^{-ry}$, where k is an arbitrary constant and is set to unity by the ordinality of the utility function. (That is, we are marking a monotonic transformation.)

[7] The full derivation is given in a supplementary mathematical appendix available from the authors. Note that the simplification in (4) derives from the fact that equalization of expected marginal utility of income implies equality of expected utility for CARA utility functions. We also assume that the firm is above the shutdown point as indicated by a sufficiently low level of alternative income.

The first order condition of (5) implies:

$$E [u' y) \cdot (pf' - y)] = 0. \tag{6}$$

(6) and A.3 together imply,

$$E [e^{-ry} (pf' - y)] = 0 \tag{7}$$

Equations (6) or (7) may be easily compared with the first order condition in the corresponding maximization problem under certainty:

$$pf' = y. \tag{8}$$

Under uncertainty, the distribution of y and the extent of risk aversion affect the outcome. This is the reason for the difference between equations (7) and (8). Following the demonstration of our Lemma 1 in the following section, we will consider this result further.

3. TWO THEOREMS ON OPTIMAL ALLOCATION AND DISTRIBUTION

In this section, we demonstrate three main results. First, we prove a lemma that, unlike the profit maximizing firm, the income per worker maximizing firm will only operate at levels for which average physical product exceeds marginal physical product. The lemma holds regardless of the production function or the degree of uncertainty and risk aversion. Next, we show that under our six assumptions, the LMF always produces a level of output for which expected income per member exceeds the value marginal product of labor. In the Illyrian model under certainty these are equated (Ward, 1958); we relate our findings to two other models in the literature which produce a result similar to ours. We derive an expression showing that the size of this inequality is proportional to price variance, risk aversion and technology, with each of these factors having separable effects on the outcome. Finally, we show that when layoffs are optimal, the expected income for working members of the EOF is greater than the sum of the compensation and the EAMEI of furloughed members, if and only if the variance of income inside the EOF is larger than the variance of the EAMEI outside the EOF.

First, we state and prove the following lemma, often demonstrated for special production functions under certainty (see Ireland and Law, 1982).

Lemma 1. Under assumptions A.1, A.2, A.3, A.4, and A.5, and the definition of average income (equation (1)), the EOF only operates at employment levels where average product exceeds the marginal product.

Proof. First consider the interior solution. From equations (1),

$$pf' + c - y = pf' + c - (pf - F - (L - l) c)/l$$

$$= p (f' - f/l) + (F + Lc)/l.$$

Equation (4) in the first order conditions implies:

$$E[u'(y) \cdot (p(f' - f/l) + (F + Lc)/l] = 0$$

or,

$$E[u'(y) \cdot p \cdot (f' - f/l)] = -E[u'(y) \cdot (F + Lc)/l].$$

or,

$$f' - f/l = -[(F + Lc)E[u'(y)]]/l E[u'(y)]$$

Thus, we conclude that $f' < f/l$.

When full employment of the membership is the optimal solution, equation (6) operates and it follows that (see the appendix):

$$E[u'(y)(p(f' - f/l) + F/l)] = 0,$$

or,

$$(f' - f/l) E[u'(y) p] = -E[u'(y)] F/l,$$

and thus again $f' < f/l$.

Using A.3, A.4, A.5, Lemma 1, and a Laplace transform or the moment generating function (see Appendix), the first order conditions may by reduced to

$$\mu_y = \mu_v + c + (\theta_p AP^2 - \theta_v)/2, \tag{9}$$

$$\mu_p MP = \mu_y - c + \theta_p AP(MP - AP), \tag{10}$$

where $\theta_p = r\sigma^2_p$, and $\theta_v = r\sigma^2_v$, are the risk factors in terms of p and v, and $\theta_p AP^2 = \theta_y$.[8]

The optimal number of working members is then given by cancelling c:

$$\mu_p MP = \mu_v + (\theta_p AP^2 - \theta_v)/2 + \theta_p AP(MP - AP). \tag{11}$$

By eliminating uncertainty terms in (9) and (10),[9] it can be seen that the corresponding deterministic model is maximized in the interior solution where:

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[8] Measures of the subjective preference towards risk and the objective level of risk always appear together algebraically; hence our definition of $\theta = r\sigma^2$ as the "risk factor".

[9] To obtain equations (12) and (13) set $\mu_y = y$, $\mu_p = p$, $\mu_v = v$, and $\theta_p = \theta_v = 0$ in equations (9) and (10).

$$y = v + c, \qquad\qquad (12)$$
$$pf' = y - c, \qquad\qquad (13)$$

and the two equations collapse into one that gives the optimal output and employment:

$$pf' = v. \qquad\qquad (14)$$

Under certainty, then, the optimal employment is determined where the value of the marginal product is equal to the expected alternative monetary equivalent income.

We now state,

Proposition 1. Under assumptions A.2 and A.3, an EOF always operates where the expected net income per worker (prior to or following compensation payments) is greater than the expected value marginal productivity of labor. This difference is related linearly to risk aversion and the variance of price.

Proof. The left hand side of equation (7) is the Laplace transform of $(pf' - y) \cdot \psi(y)$ evaluation at r, where $\psi(y)$ is the distribution of y, corresponding to the distribution $\phi(p)$, given by A.4. Making use of the Laplace transform or the moment generating function and some additional calculations (see the mathematical appendix), equation (7) leads to:

$$\mu_p MP = = \mu_y + \theta_p AP \, (MP - AP) \qquad\qquad (15)$$

Thus, $\mu_y > \mu_p MP$ since $MP < AP$. Similarly, for the nonbinding case, from equation (10) we have,

$$\mu_y > \mu_p MP + c > \mu_p MP \qquad\qquad (16)$$

The proposition follows from the application of Lemma 1. As shown in the appendix, it holds whether income per capita is defined net or gross of compensation (and so holds in interior or corner solutions).

In each case, the expectation and variance of price will affect the choice but compensation and the mean and variance of outside EAMEI will play no role in the solution.

Equations (10) and (15) are convenient for econometric testing of the theory of the expected utility maximizing LMF under price uncertainty and full employment of its membership. This would be a natural extension of recent applied studies of Italian and other OECD EOFs. In particular, Smith (1984) found Italian producer cooperatives and American employee owned plywood firms operating at value marginal products considerably below average member incomes (net of capital costs), which may be explained alternatively by (a) risk aversion, (b) an objective function which assigns positive weight to employment as well as income (LMF associations state that this is a goal), or (c) dynamic allocation (Barlett (1987); Smith and Ye (1988)). These sources have different allocational implications.

Corollary 1. The income per worker maximizing firm always produces where $AP < \mu_p/\theta_p$.

This clearly suggests that the EOF is more likely to exist under higher expected price and lower price variance. Proposition 1 indicates that the EOF is likely to produce farther down the AP schedule; and this is also reflected in the form of the corollary. The corollary is readily testable using standard panel data sets used in the empirical literature on the EOF along with industry price data.

We now state:

Proposition 2: $E(y) \geq E(v + c)$ iff $Var(y) \geq Var(v + c)$,
    where $y = (pf - F - (L - 1) \cdot c)/l$.

Proof. The proposition follows immediately from equation (9).

Expressed another way, the theorem states that the expected income for working members is greater than the sum of the compensation and the EAMEI for furloughed members, if and only if the fluctuation of income inside the EOF is larger than the fluctuation of the monetary-equivalent value outside the EOF. This finding is to be expected given the risk-averse decision-making process we have modeled. But it is at variance with the previous literature.

Among other things the proposition tells us that the expected income of working members is equal to the EAMEI plus compensation to laid off members if and only if all members are risk neutral. This follows from the fact that the probability that $\sigma^2_y = \sigma^2_v$ is zero. Thus the results in earlier state-contingent models carry over to our more realistic treatment of uncertainty only if workers are risk neutral and/or alternative monetary equivalent income is given with certainty.[10]

4. COMPARATIVE STATICS FINDINGS

Comparative statics on the equilibrium given by equation (9) and (10) the interior solution) provide interesting insights into the questions raised in the introduction. They also allow for a more complete comparison between our model and earlier treatments of the EOF facing uncertainty in the literature, including Illyrian models and state contingent approach to layoff compensation. The main results are summarized in Table 1.

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[10] For example, Brewer and Browning (1982) assumed that outside income was given with certainty, reasonable given that the rest of the analysis also assumed certainty. McCain (1985) assumes that prices are uncertain but outside income is certain. Miyazaki (1984) and Miyazaki and Neary (1986) state that outside utility is uncertain; their results do not generalize beyond state-contingent contracts unless risk neutrality is to be assumed. With an ex ante contract risk-averse members will optimally equalize the *expected* marginal utility (instead of simply the marginal utility) of income across employed and unemployed states.

Consider first the effects of changes in parameters on unemployment compensation. There is no compensation paid to furloughed members in the Illyrian literature. In our model, compensation increases in expected price and decreases in EAMEI, which parallels findings in the state-contingent literature (Miyazaki, 1984), as well as expected income maximization among members under certainty. We also find that compensation decreases in price variance and increases in the variance of EAMEI. This finding corresponds to economic intuition and yet does not follow from the earlier models, which assume away either member property rights or the problem of asymmetric information and transaction costs detailed in Section 2. All of these results correspond to the logic of expected utility maximizing behavior of LMF members.

The effects of an increase in the members' risk aversion on unemployment compensation is complex. The exercise may be interpreted as a comparison of two firms, identical in all respects except for risk aversion. It may also be reasonably interpreted as examining the results of a change in the composition of membership (for example, the new median member may be more risk averse than the previous median member). Following a "rise in risk aversion", then, compensation will rise if the variance in alternative income exceeds the variance in employment income. Conversely, compensation will fall if the opposite inequality holds and remain unchanged if the two variances are identical. Thus a more risk averse membership will move to rise expected incomes for those members in the riskier position at the expense of expected incomes for those in the less risky position. This appealing finding is also unique to our approach. The Illyrian LMF literature ignores the issue and the state contingent contract literature assumes it away by asserting that income variance can be made equal for working and nonworking members.

Next we consider the effects of changes in the parameters on the optimal number of working members. Working employment should increase with expected price; any other finding would be very surprising, given the earlier literature. (For example Bonin (1981), Svejnar (1982) and Miyazaki and Neary (1983); this result follows from our equations (9) and (10)). Mathematically, in the present case this result depends on a positive sign for $D = \theta_p (MP - AP)^2 + lf'' (\theta_p AP - \mu_p)$. A negative sign cannot be ruled out technically, given a sufficiently small output price and a sufficiently high level of risk aversion and or price variance. But economically speaking, these conditions are clearly suggestive of a below-shutdown point for the firm.[11] Moreover, for a number of other comparative static results, a negative sign on D leads to nonsensical results.

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[11] Note that there is nothing in our model which constrains workers to pay the firm's fixed cost even if the firm declares bankrupcy, that is, the firm is not an unincorporated partnership. Under this constraint, the results for the case in which D is negative become more relevant. In particular, in this case the Illyrian effect may return for a sufficiently high price risk factor. Recall that the unemployment income risk factor is held constant in this analysis. If the price risk factor is very high relative to the unemployment income risk factor, employment may fall

Employment of members increases when the variance of EAMEI rises if and only if $D > 0$; employment falls with an increase in EAMEI only under the same condition. The latter result is expected from the case of certainty (see again our equations (12) and (13)), and, ceteris paribus, no other result makes sense under uncertainty.

Next, consider the ambiguous effect of an increase in the variance of price (or alternatively of risk aversion) on employment. As seen in the Table, its sign depends on the difference between the average product and twice the marginal product. If this difference is negative, then the employment response is negative; but the opposite holds when this difference is positive (allowing that $D > 0$). The same effect is found in the risk-averse profit maximizing firm. But when the difference is positive (which corresponds to a large firm, i.e. one with a large total membership) an increase in price variance may lead the firm to wish to spread the risk and reduce compensation commitments by increasing output through the employment of more of its members.

The effect on employment of an increase in risk aversion is also ambiguous. Some insight is gained by reference to a special case in which income and EAMEI variances are equal, $\sigma^2_y = \sigma^2_v = \sigma^2$ In this case,

$$\partial l / \partial r = \sigma_2 (1 - MP/AP)/D > 0,$$

since $MP < AP$ (Lemma 1) and $D > 0$. With $AP < 2MP$ and σ^2_p sufficiently greather than σ^2_v, the opposite sign may obtain. That is, for a sufficiently high price risk relative to EAMEI risk the members may vote to contract employment when they become (at the median, to offer an interpretation) more averse to risk.

The comparative static results of changes in fixed costs on the LMF has been widely studied since Ward (1958). Our finding are intuitive but different from those resulting from Illyrian models of the LMF. We find that an increase in these costs will decrease compensation by $1/L$ times the incremental change (all members share these costs equally). Further, such an increase will have no effect on working membership (it has the effect of a lump sum tax on each member, working or furloughed).

Finally, consider the effects of changes in membership, perhaps best interpreted as a comparison of EOFs identical in every respect but the total number of members. The number of working members, which is chosen on efficiency grounds, will remain unchanged. Indeed, note that the membership level does not appear in the optimality

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when expected price rises because the members can afford to take more advantage of the relatively riskless unemployment environment. In this case, too, employment may increase when the EAMEI rises. The firm can now afford to employ more of its members because compensation payments can be decreased. The members now act to spread the higher intrafirm (price) risk more widely by expanding output. It is difficult to find an analogous argument as to why even the fully liable LMF membership should even wish to employ less members after unemployment risk rises.

conditions (9) and (10). The change in compensation for a change in membership is given as —c/L; in other words the compensation elasticity of employment is —1.

<div align="center">Table one</div>

$$\partial c / \partial \mu_p = lAP/L > 0,$$

$$\partial c / \partial \mu_v = -l/L < 0,$$

$$\partial c / \partial \sigma^2_p = -rlAP^2/2L < 0,$$

$$\partial c / \partial \sigma^2_v = rl/2L < 0,$$

$$\partial c / \partial r = -l\,(\sigma^2_y - \sigma^2_v)/2L \quad \begin{cases} < 0 \ \ if \ \sigma^2_y > \sigma^2_v, \\ > 0 \ \ if \ \sigma^2_y < \sigma^2_v, \end{cases}$$

$$\partial l / \partial \mu_p = lMP/D > 0,$$

$$\partial l / \partial \mu_v = -l/D < 0,$$

$$\partial l / \partial r = l\,[(\sigma^2_v - \sigma^2_y)/2 + \sigma^2_p AP\,(AP - MP)]/D,$$

$$\partial l / \partial \theta_p = lAP\,(AP - 2MP)/2D,$$

$$\partial l / \partial \theta_v = l/2D.$$

where $D = \theta_p\,(MP - AP)^2 + lf''\,(\theta_p AP - \mu_p).$

5. CONCLUSIONS AND EXTENSIONS

In this paper, we have argued that a noncontingent contract approach is the more appropriate framework for examining the allocational and distributional behavior of the employee-owned firm facing uncertainty not only over output price but also over the expected alternative monetary equivalent income. The approach has provided a more realistic and economically justifiable theory of the contracts that will be signed among members who hold some form of "property rights" in the firm. Moreover, it has provided a new set of basic results answering a number of important questions which were assumed away in the earlier literature.

A number of extensions of the analysis suggest themselves. Perhaps the most important future extension would be to develop the model in the context of Sertel's (1982, 1987) general equilibrium partnership market.[12] Sertel's model examined equilibrium under certain-

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[12] The authors would like to thank an anonymous referee for this suggestion.

ty—examining only the first moment (or mean return). Our paper has laid the foundation for an extension of Sertel's partnership equilibrium notion to the second moment, measuring comparative risks faced by workers under wage and various labor-management contracts.

While we can only introduce this subject in the present paper, the main features of equilibrium determination under Sertel-type contract markets for employment in either employee or investor owned firms may be characterized as follows. In investor-owned firms, the wage rate w is low but stable, while workers face a probability, ρ, of being laid off. Unemployment compensation, α, is relatively low. In employee-owned firms, higher-average income μ_y fluctuates according to its variance σ^2_y. Income is adjusted for amortization of any partnership fees or other entrance costs. When laid off, according to the model in Section 3, these employee owners receive the same income as working employee owners, adjusted for the value of leisure or second-best employment. Using the Markowitz mean-variance utility function, $u = u(\mu_y, \sigma^2_y)$, the equilibrium across types of firms will be:

$$\rho u(\alpha, o) + (1 - \rho) u(w, o) = u(\mu_y, \sigma^2_y). \qquad (17)$$

Mean-variance equilibrium is thus achieved among EOFs and across to investor-owned firms. Any other utility advantages to working in an EOF would lead to the adjustment of the right-hand-side of (17) accordingly.

Secondly, in the paper we have assumed that all members are identical. In practice, members may differ one from another in a number of relevant ways, including the extent of risk aversion and the individual's productivity. On particularly salient dimension over which members may differ is in the level and variance of their expected alternative monetary equivalent income. Formally, extensions addressing the issues associated with nonidentical members involve modifying assumption A.3. It is relatively easy to allow for two types of member, identical in all but a very limited number of characteristics, assuming an objective such as the maximization of the sum of cardinal utilities.

Consider first that some members face a higher alternative income variance, a lower average alternative income, or both. If productivity is not lower for these members (for example, they face discrimination), we would expect the EOF to be more likely to lay off workers with better outside prospects first, because the cost (c) of doing so would be lower. This practice would result in a higher expected utility for all members. To illustrate, if female members face discrimination in the labor market, male members would be laid off first. Such results require that we adjust A.6 to keep it in accord with the modification of A.3.

In some cases, there may be a more complicated relationship between productivity and the mean and variance of alternative income. For example, in many labor markets, older workers receive higher incomes if they find alternative employment, but may search longer for a job and face higher probability of long-term unemployment. First, the mean-variance tradeoff in alternative income would have to

be accounted for. Then, assuming younger and older members contribute the same marginal product, a strategy of laying off first the members with higher outside expected utility would maximize expected utilitiy per member. This strategy would need no modification if the same members contributed the same or a smaller marginal product. But if the members with higher EAMEI also contributed a higher marginal product within the EOF, the decision rule on whom to lay off first would be based on a more complicated tradeoff between inside and outside contributions to expected income.

In other cases, members might have different degrees of risk aversion. If one group has a lower variance of outside income, but also has greater risk aversion, this additional tradeoff must be considered in determining the optimal order for layoffs.

In another extension, we may consider that potential members face two kinds of risk, that in the job market and that in capital market. The standard literature on the EOF under uncertainty has focussed on the problem of capital market risk. To the extent EOFs are internally financed, members must bear more risk than nonmembers or employees of conventional firms, who have greater opportunities to diversify their capital portfolios through financial intermediaries (Neuberger and James (1973); Bonin and Putterman (1987)). A number of risk-pooling strategies are practiced by EOF associations (Smith and Ye (1987)). Beyond this, workers who become members of EOFs tend to start with relatively little wealth, and have little opportunity for portfolio diversification. In relative terms, other sources of risk, especially the risk of unemployment, will rationally be ranked as more important by workers, for such workers receive most of their income from labor. Such risks are to a large degree captured in the mean and variance of alternative income as modeled in the paper. Thus one useful extension would be to consider explicitly the conditions under which job market risk dominated capital market risk in decisions, quite independent from inter-enterprise risk pooling strategies.

This analysis could be developed to show how workers with higher job market risk are more likely to become members of EOFs. If the workers face higher job market risk due to "lemons" problem (Akerlof, 1971), in which conventional firms are reluctant to hire certain types of employees, the EOF may even help to solve this problem. Presumably, information about the productivity and effectiveness as a coworker may be more accessible to fellow members or potential members than to management. These extensions further clarify the underlying causes of the explosive growth in.

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MATHEMATICAL APPENDIX

A. DERIVATION OF EQUATION (15)

Equation (15) can be derived by either a Laplace transform or a moment generating function; we present a derivation making use of the former. Consider first the case of full employment (hence no compensation payments).

Equation (15) is derived from equations (5) and (7), making use of A.4 and by applying a Laplace transformation or the moment generating function. Solving equation (5) for p, we have

$$p = (yl + F)/f \qquad\qquad\qquad \text{(M.1)}$$

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* See *CRC Standard Mathematical Tables*. 26th ed. Edited by William H. Beyer. Boca Raton, Florida: CRC Press, Inc., 1983.

Subtituting (M.1) into (7) yields:

$$E\left[e^{-ry}\right] f'F/f - E\left[ye^{-ry}\right]\left(\mu - f'\partial/f\right) = 0. \tag{M.2}$$

To shorten the notation, let

$$A = f'F/f, \text{ and } B = \mu - f'l/f; \tag{M.2}$$

Then (M.2) becomes

$$AE\left[^{-ry}\right] = BE\left[ye^{-ry}\right]. \tag{M.4}$$

We know that p has a truncated normal distribution with μ_p and σ^2_p by A.4, so y must have a normal distribution, $\varphi(y)$, with mean μ_y variance σ^2_y where

$$\mu_y = (\mu_p f - F)/l \text{ and } \sigma^2_y = \sigma^2_p f^2/l^2. \tag{M.5}$$

Our next task is to calculate $E\left[ye^{-ry}\right]$, which is simply the Laplace transform of $y\varphi(y)$ evaluated at r. It can be shown (see section c of this appendix) that

$$E\left[ye^{-ry}\right] = \mu_y E\left[e^{-ry}\right] - r\sigma^2_y E\left[e^{-ry}\right]. \tag{M.6}$$

Plugging (M.6) into (M.4), and cancelling $E\left[e^{-ry}\right] \neq 0$, we end up

with

$$A - B\mu_y + Br\sigma^2_y = 0. \tag{M.7}$$

Applying (M.3) and (M.5) to (M.7), and recalling that we defined $\theta_p = r\sigma^2_p$, we get equation (15):

$$\mu_p MP = \mu_y + \theta_p AP (MP - AP),$$

where $AP = f/l$ and $MP = f'(l)$ are the average and marginal products.

B. NEXT CONSIDER THE INTERIOR SOLUTION

By A.2, we must have (see below)

$$u(\omega) = -e^{-r\omega},$$

thus

$$u'(\omega) = re^{-r\omega}.$$

So equations (3) and (4) are reduced to

$$E\left[e^{-ry}\right] = E\left[e^{-r(v+c)}\right] \tag{M.8}$$

and

$$E[^{-ry} \cdot (pf' + c - y)] = 0. \qquad \text{(M9.)}$$

First, for (M.8), we find that $E[e^{-r\omega}]$ is the moment generating function of ω evaluated at $(-r)$; hence, as ω is normally distributed, with $E[\omega] = \mu$ and $v(\omega) = \sigma^2$,

$$E[e^{-r\omega}] = e^{-r\mu + r^2\sigma^2/2}.$$

Therefore (M.8) is simplified, by A.3 and A.4, to:

$$-r\mu_y + r^2\sigma^2_y/2 = -r(\mu_v + c) + r\sigma^2_v/2,$$

which, recalling $\theta_p = r\sigma^2_p$, $\theta_v = r\sigma^2_v$ and $\sigma^2_y = \sigma^2_p f^2/l^2$, gives our result.

Next, let us calculate (M.9). The calculation involved here is very similar to the one we used in Section A of the mathematical appendix, but now the definition of y includes compensation payments. Solving for p, we have

$$p = (yl + F = (L - l) \cdot c)/f. \qquad \text{(M.10)}$$

Substituting (M.10) into (M.9),

$$E[e^{-ry}](f' \cdot (F + (L - l) \cdot c)/f + c) = E[ye^{-ry}](1 - f'l/f). \qquad \text{(M.11)}$$

To shorten the notation, let

$$A = f' \cdot (F + (L - l) \cdot c)/f + c, \text{ and} \qquad \text{(M.12)}$$

$$B = 1 - f'l/f. \qquad \text{(M.13)}$$

Then (M.11) becomes

$$AE[e^{-ry}] = BE[ye^{-ry}]. \qquad \text{(M.14)}$$

Since p has a truncated normal distribution with μ_p and σ^2_p by A.4, y must have a normal distribution $\varphi(y)$ with μ_y and σ^2_y where

$$\mu_y = (\mu_p f - F - (L - l) \cdot c)/l \qquad \text{(M.15)}$$

and

$$\sigma^2_y = \sigma^2_p f^2/l^2. \qquad \text{(M.16)}$$

Hence $E[ye^{-ry}]$ is the Laplace transform of $y\varphi(y)$ evaluated at r. It can be shown (see Section c below) that

$$E[ye^{-ry}] = \mu_y E[r_{-ry}] - r\sigma^2_y E[e^{-ry}]. \qquad \text{(M.17)}$$

Plugging (M.17) into (M.14), and cancelling $E\,[e^{-ry}] \neq 0$, we end up with

$$A - B\mu_y + Br\sigma^2_y = 0. \qquad\qquad (M.18)$$

Applying (M.12), (M.13), (M.15), and (M.16) to (M.18), and recalling $\theta_p = r\sigma^2_p$, we obtain equation (10):

$$\mu_p MP = \mu_y - c + \theta_p AP\,(MP - AP)$$

where $AP = f/l$ is the average product and $MP = f'(l)$ is the marginal product.

C. DERIVATION OF EQUATIONS (M.6) AND (M.17)

We have

$$E\,[ye^{-ry}] = \int_{-\infty}^{\infty} e^{-ry}\, y\varphi(y)dy \qquad\qquad (M.19)$$

where $\varphi(y) = \exp\,(-y - \mu_y)^2/2\sigma^2_y)/\sqrt{2\pi}\sigma$.

Let $x = (y - \mu)/\sqrt{2}\,\sigma$, then $y = \sqrt{2}\,\sigma x + \mu$ and $dy = \sqrt{2}\,\sigma dx$.

Equation (M.19) becomes

$$E\,[ye^{-ry}] = 1/\sqrt{2\pi}\sigma \cdot \int_{-\infty}^{\infty} (\sqrt{2}\sigma x + \mu)\, e^{\sqrt{2}r\sigma x - r\mu} \cdot e^{-x^2} \cdot \sqrt{2}\,\sigma dx$$

$$= 1\sqrt{\pi} \int_{-\infty}^{\infty} (\sqrt{2}\,\sigma x e^{-x^2 - \sqrt{2}r\sigma x - r\mu} + \mu e^{-x^2 - \sqrt{2}\sigma x - r\mu})\,dx$$

However,

$$e^{-x^2 - \sqrt{2}r\sigma x - r\mu} = e^{-(x^2 + 2\sqrt{2}r\sigma x/2 + r^2\sigma^2/)} \cdot e^{-r\mu + r^2\sigma^2/2}$$

$$= e^{-(x + r\sigma/\sqrt{2})^2} \cdot E\,[e^{-ry}].$$

Since $E\,[e^{-ry}]$ is simply the moment generating function of y evaluated at $(-r)$,*

$$E\,[e^{-ry}] = e^{-r\mu + r^2\sigma^2/2}.$$

So,

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* See *SRC Standard Mathematical Tables*. 26th ed. Edited by William H. Beyer. Boca Raton, Florida: CRC Press, Inc., 1983.

$$E[ye^{-ry}] = E[e^{-ry}]/\sqrt{\pi} \cdot \int_{-\infty}^{\infty} (\sqrt{2}\,\sigma)\, xe^{-(x+\sigma/\sqrt{})^2}\, dx$$

$$+ \mu E[e^{-ry}]/\sqrt{\pi} \cdot \int_{-\infty}^{\infty} e^{-(x+r\sigma/\sqrt{2})^2}\, dx.$$

Let $z = x + r\sigma/\sqrt{2}$. Then $x = z - r\sigma/\sqrt{2}$ and $dx = dz$, and we have

$$E[ye^{-ry}] = \sqrt{2}\,\sigma E[e^{-ry}]/\sqrt{\pi} \cdot \int_{-\infty}^{\infty} (z - r\sigma/\sqrt{2}) \cdot e^{-z^2}\, dz +$$

$$\mu E[e^{-ry}]/\sqrt{\pi} \int_{-\infty}^{\infty} e^{-z^2}\, dz$$

But

$$\int_{-\infty}^{\infty} e^{-z^2}\, dz = \sqrt{\pi} \quad \text{and} \quad \int_{-\infty}^{\infty} ze^{z^2}\, dz = 0.^*$$

Hence,

$$E[ye^{-ry}] = \mu E[e^{-ry}] - r\sigma^2 E[e^{-ry}].$$